An Empirical Model of Consumer Affiliation and Dynamic Price Competition*

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Abstract
We study the pricing decision of firms when consumers may become “affiliated” with previously purchased products. Affiliation can arise from habit formation, brand loyalty, or switching costs, and it has important implications for the interpretation of equilibrium outcomes and counterfactual analysis. We analyze the effect of affiliation in the context of mergers, and we show that a (misspecified) static model will predict price effects larger than those generated by the dynamic model. The dynamic incentive to invest in future demand tends to suppress post-merger price increases. We develop an empirical model of dynamic demand that can be estimated independently from supply-side assumptions and uses only market-level data. By estimating the model without supply-side assumptions, we are able to test for forward-looking behavior by firms. Using a hypothetical merger of two major retail gasoline companies that anticipate habit formation by their consumers, we find that a static model predicts a price increase substantially greater than the price increase predicted when accounting for dynamics.

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1 Introduction

Consumers are often more likely to buy a product if they have purchased it previously. This tendency reflects both exogenous preferences and state-dependent utility that is affected by past behavior (Heckman, 1981). Consumer state dependence, which we call “affiliation,” may arise from habit formation, brand loyalty, or switching costs. There is a rich empirical literature that establishes the presence of such affiliation in a variety of markets.\(^1\) This behavior is typically identified by examining the choice patterns of individual consumers over time.

In this paper, we develop an empirical model of dynamic, product-specific consumer affiliation that can be estimated using aggregate market-level data. To disentangle heterogeneity in preferences from state dependence arising from previous purchases, we impose a demand system and rely on the panel structure of our data. Intuitively, after adjusting for cross-sectional fixed effects, consumer affiliation is captured by the residual correlation in shares over time (and their relation to prices). We consider the implications of this model on the pricing behavior of firms, and, using a counterfactual merger simulation, compare the predictions of the dynamic model to those of a static model.

In response to consumer affiliation, profit-maximizing firms will internalize the effect of the current price on the distribution of affiliated consumers in future periods. This will, in general, lead firms to invest in future consumers by lowering the price relative to the static optimum. Even with myopic consumers, the model predicts two pricing patterns that are often observed in the real world but inconsistent with static models: (1) prices are slow to adjust to changes in marginal costs, and (2) prices anticipate expected changes in future costs. Thus, dynamic consumer behavior offers an explanation for why prices may be slow to adjust over time. Prices in a wide array of industries have been documented to react slowly to cost changes. Grocery stores (Peltzman, 2000), banking (Neumark and Sharpe, 1992), and retail gasoline (Borenstein et al., 1997) are just a few examples of markets where firms gradually pass through marginal cost shocks to consumers. While slowly adjusting prices are an important indicator of dynamics in these markets, the demand estimation literature has largely ignored this property.

We examine the implications of dynamic pricing incentives in the context of horizontal mergers. Antitrust authorities will challenge a merger if the merging firms are expected to increase the price of a product by a significant amount, which is often taken to be 5 percent. The ability to accurately predict the price effect of a merger hinges on an appropriate representation of the firms’ pricing incentives. In the presence of consumer affiliation, a first-order determinant of price is its effect on future demand. A static model, which omits this incentive, will incorrectly attribute this effect to consumer elasticities and the degree of competition. As we demonstrate, static models consistently over-predict the price effects of a merger, compared to an analysis that accounts for the dynamic incentive.

Antitrust authorities often analyze mergers in markets that are likely to be characterized by consumer affiliation. For example, in its lawsuit against Swedish Match-National Tobacco, the Federal Trade Commission (FTC) cited strong brand loyalty as a barrier to entry.\(^2\) Similarly, the US Depart-

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\(^1\)See the related literature section for examples.

ment of Justice cited customer switching as an important factor in its case against the UPM-MACTac merger. In defense of its acquisition of TaxACT, H&R Block cited the importance of dynamic incentives in exerting downward pricing pressure post-merger (Remer and Warren-Boulton, 2014). Both the FTC and DOJ routinely investigate mergers in consumer products markets, where inertia in brand choice has been frequently documented. Yet, perhaps due to computational complexity and compressed investigative timelines, dynamics are seldom directly modeled in when analyzing unilateral competitive effects. We develop a model that significantly reduces the computational and data burden of estimating the impact of these dynamics.

We apply the model using data from retail gasoline markets, where pricing patterns consistent with consumer brand loyalty have been documented (Borenstein (1991), Hastings (2004), Doyle et al. (2010)). Furthermore, retail gasoline has a direct link to current antitrust concerns. In June and December of 2017, the FTC challenged Alimentation Couche-Tard’s acquisitions of Empire Petroleum Partners and Holiday, respectively, on the basis of overlapping retail gasoline stations in a number of states. The FTC argued that the merger with Holiday would reduce the number of independent competitors from four to three in five markets and from three to two in five other markets. Similar arguments were made for the merger with Empire Petroleum, which lead to the divestiture of 71 retail gas stations. The 4-to-3 and 3-to-2 heuristics are commonly cited by the antitrust authorities, and they are primarily informed by the logic of static models.

The demand model we introduce is a straightforward extension of the standard discrete-choice logit model with myopic consumers. In contrast to the typical random-coefficients model, we allow the distribution of unobserved heterogeneity to be state-dependent, affected by past purchase behavior. We restrict the random coefficients to affect a single product for each consumer type, corresponding to our notion of product-level affiliation. The model can be estimated using aggregate market-level shares and prices, which is the typical data used in demand estimation and merger simulation. Importantly, the model can also be estimated independently of supply-side assumptions, and we can therefore use the estimated demand model to test for forward-looking behavior by firms.

To provide intuition about the model, we develop theoretical results for the optimal monopoly price and use numerical simulations in a duopoly setting. In the steady state, we show that optimal prices may either be higher or lower in the dynamic model compared to a static model, depending on the relative price sensitivity of affiliated and unaffiliated consumers. Thus, optimal firms may either “invest” or “harvest” in perpetuity, although both incentives matter in the steady state.

We first consider mergers in the context of the model using numerical simulations in a duopoly setting. We demonstrate that the presence of affiliation in duopolistic competition may increase prices more than a merger to monopoly would in a static model. Second, we show that the relative price effect of a merger has non-monotonic patterns, which is relevant for the consideration of antitrust authorities. The percent price effect of a merger may either increase or decrease with the

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3Ibid
4For example, Dubé et al. (2010) estimate state dependence in consumers’ choices of orange juice and margarine, where the root cause is brand loyalty developed through consumption.
7Absolute prices increase monotonically with the degree of affiliation.
probability that consumers develop an affiliation. When consumers are sufficiently price insensitive, then affiliation allows firms to capture a large portion of the monopoly rents pre-merger, resulting in a smaller increase in post-merger prices.

Next, we consider the empirical implications of failing to account for dynamic demand in a merger analysis. In this exercise, we calibrate a static demand model to data generated by the dynamic model and perform a merger simulation. Compared to the true impact of the merger, the (misspecified) static logit model systematically over-predicts merger price effects. In the dynamic model, the incentive to invest in future demand pushes prices down, and this effect remains after the merger. The static model falsely attributes a portion of this incentive to competition, which disappears post-merger, resulting in higher prices.

The model can be taken to data, which we demonstrate through an analysis of retail gasoline markets. We advance the empirical literature by developing an estimation method that accommodates dynamic demand but does not require the assumptions used in standard dynamic estimation techniques, such as those developed by Bajari et al. (2007) and Pakes et al. (2007), to identify the dynamic parameters. Importantly, we do not require supply-side assumptions about the competitive game or the expectations of firms to estimate demand. As supply-side behavior depends crucially on whether future shocks are expected or unexpected, we view this as a significant advantage.

The data needed to estimate the model are commonly used in the demand estimation literature, namely, prices, shares and cost shifters (or, alternatively, product characteristics). Nonetheless, we show that with firm-level shares (across all consumer types), it is possible to separately identify each firms’ current shares of affiliated and unaffiliated consumers, as well as the probability that a consumer becomes affiliated after purchasing. Thus, we allow for endogenous unobserved heterogeneity through the presence of a serially correlated state variable for each firm. This flexibility has traditionally been a challenge for the estimation of dynamic models.

We estimate the model using a rich panel data set of prices, shares, and costs for retail gasoline stations. In this context, the model is best interpreted as one of habit formation, wherein some consumers return to the gas station from which they previously purchased without considering alternative sellers. We find evidence of strong demand dynamics. We estimate that 61 percent of consumers become affiliated to the brand from which they previously purchased on a week-to-week basis, and therefore effectively do not consider competitors. On average, 99 percent of affiliated consumers re-purchase from the previous brand, whereas only 38 percent of consumers that are not affiliated purchase gasoline. Affiliated consumers display a much lower price sensitivity. Unaffiliated consumers have an average elasticity of -7.0, whereas affiliated consumers have a near-zero average elasticity. Though unaffiliated consumers are a minority of all consumers that purchase gasoline, they are important for disciplining prices in equilibrium.

To highlight the importance of accounting for dynamics when predicting firm behavior, we impose a supply-side model of price competition. In contrast to the literature, we impose relatively weak assumptions about supply-side behavior in order to conduct counterfactual analysis. From the estimated demand model, we obtain the derivative of static profits, which we use to infer the dynamic component of the firms’ first-order conditions. We project these estimates onto state variables to construct a reduced-form approximation to the dynamic pricing incentives. Using this approx-
imation, we perform a merger analysis between two major gasoline retailers and re-compute the price-setting equilibrium in each period. With the dynamic model, we estimate that prices would increase by 2.1 percent post-merger. A static model, on the other hand, predicts a 5.9 percent price increase. The price effect predicted by the static model would likely receive antitrust scrutiny, whereas the smaller effect found by the dynamic model would be less of a concern. The merger increases both the static incentive to increase prices and the dynamic investment incentive to lower prices. Thus, the dynamic incentives serve to mitigate the static effects that arise from a merger.

Prior to estimating the model, we present reduced-form evidence of dynamic demand and dynamic pricing in retail gasoline markets. Consistent with investment in affiliated consumers, we find that new entrants initially price lower than established firms. We then examine cost pass-through. Using the data to separate out expected and unexpected costs, we show that firms respond differentially to these two measures. Importantly, firms begin raising prices in anticipation of higher costs approximately 28 days prior to a cost shock. Firms exhibit “full” pass-through for expected cost changes, raising prices by 1.02 dollars for each dollar increase in cost. Conversely, pass-through of unexpected costs is limited, with firms raising prices by only 0.66 cents for an unexpected dollar increase in costs. We explore heterogeneity in firm pass-through and how it relates to the competitive conditions in the market.

Related Literature

Our analysis builds on existing theoretical work on dynamic price competition when consumers are habit-forming or have switching costs. These features link directly to our notion of consumer affiliation. For examples, see Farrell and Shapiro (1988), Beggs and Klemperer (1992), and Bergemann and Välimäki (2006). There are alternative strategic reasons for dynamic pricing, including experience goods (Bergemann and Valimaki, 1996), network effects (Cabral, 2011), learning-by-doing (Besanko et al., 2017), and search (Stahl, 1989). We add to this literature by considering the effect of consumer dynamics on post-merger price incentives.

There is a significant empirical literature that documents state dependence in consumer preferences using consumer-specific purchase histories. Meaningful switching costs, due to brand loyalty or consumer inertia, have been found in consumer packaged goods (Shum, 2004; Dubé et al., 2010), health insurance (Handel, 2013), and auto insurance (Honka, 2014). An important contribution of this paper is that we are able to estimate the magnitude of these switching costs, or consumer affiliation, using market-level data on quantities rather than more detailed consumer-specific data. Aggregate market-level data is more prevalent, especially when considering settings of inter-firm competition. Like Dubé et al. (2009), we consider the implications of state dependence on the pricing behavior of firms.

We contribute to a growing body of empirical models of dynamic demand. Existing work focuses on different contexts that drive dynamic behavior. Hendel and Nevo (2013) consider a model with storable goods and consumer stockpiling. Gowrisankaran and Rysman (2012) and Lee (2013) consider the purchase of durable goods with forward-looking behavior by consumers. In contrast

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We what we mean by dynamic pricing is that there are intertemporal spillovers. This should not be confused with static pricing in response to changing market conditions, which is often colloquially referred to as “dynamic pricing.”
to these papers, we focus on settings with positive dependence in purchasing behavior over time. The literature highlights the issue, common to our setting, that misspecified static models will produce bias elasticities. Hendel and Nevo (2013) point out that this will matter in a merger analysis. We complement this point by providing a case in which the dynamic incentives, rather than biased elasticities, are the primary source of concern in model misspecification.

We propose a reduced-form method to approximate the dynamic incentives in supply-side pricing behavior, which allows us to side-step some of the challenges present in the estimation of dynamic games. Compared to value-function approximation methods proposed by Bajari et al. (2007) and Pakes et al. (2007), we rely more heavily on the structure of the demand model and place weaker assumptions on supply-side behavior. Our focus on the pricing behavior of firms precludes the use of several developments in the conditional choice probabilities literature, which relies on discrete actions (e.g. Aguirregabiria and Mira (2007), Arcidiacono and Miller (2011)). For a helpful summary of some developments in the estimation of dynamic games, see Aguirregabiria and Nevo (2013).

Finally, we contribute to the literature on cost pass-through, in general, and in retail gasoline markets, specifically. Weyl and Fabinger (2013) demonstrates the valuable information embedded in firm marginal cost pass-through rates, such as tax incidence and the welfare consequences of third-degree price discrimination. Miller et al. (2015) shows that cost pass-through estimates can be used to predict the price effects from mergers. In this article, we find that estimates of cost pass-through that don't account for anticipatory price responses can yield biased estimates in markets where dynamic demand is present. We also contribute to the literature that estimates dynamic price adjustments in retail gasoline markets (Borenstein et al. (1997), Lewis (2011)). Borenstein and Shepard (1996) shows that tacitly colluding firms will decrease price with expected future cost changes, and find support for this behavior using retail gasoline data. We show that competing firms will increase current prices in response to a future positive cost shock when consumers are prone to habits or brand loyalty, and find strong support for this pricing pattern in the data. We therefore show that anticipatory price responses are an equilibrium outcome in non-collusive markets.

2 A Model of Oligopolistic Competition with Dynamic Demand

We develop a dynamic model of oligopolistic competition with product differentiation where consumers may become affiliated with the firm from which they purchased previously. Affiliation may be interpreted as habit formation, brand loyalty, or switching costs. Consumers in the model are myopic in that they maximize current period utility rather than a discounted flow of future utility. This assumption is likely a good fit for retail gasoline markets, where consumers do not choose a gas station anticipating that it will limit their future choice set; rather, some consumers are likely to return to the same gas station due to habit-formation or brand loyalty.

As detailed below, we introduce consumer dynamics by allowing for endogenous unobserved heterogeneity in a differentiated product demand model. We then place the demand model into a dynamic oligopoly setting. Even though consumers are myopic, key dynamics arise when firms in-
ternalize the effect of sales today on future profits through the accumulation of affiliated consumers. We provide intuition for this model via a theoretical analysis of a monopolist, and use numerical simulations to examine the post-merger price incentives in a duopoly setting.  

2.1 Demand

We extend the standard logit discrete choice model to allow for unobserved heterogeneity that depends on past purchases. The first assumption below presents a random coefficients utility formulation with myopic choice. The second assumption restricts the random coefficients so that the type-specific utility shock affects only a single product, corresponding to a habit-formation setting. The third assumption places restrictions on the evolution of consumer types over time.

Assumption 1: Myopic Discrete Choice Consumers in each market select a single product \( j \in J \) that maximizes utility in the current period, or they chose the outside good (indexed by 0). For the empirical model, we make the additional assumption that utility follows the random-coefficient logit setup. Consumers are indexed by discrete types \( i \in I \), and we allow for the distribution of types to change endogenously over time.

A consumer \( n \) of type \( i \) receives the following utility for choosing product \( j \):

\[
 u^{(n)}_{jt}(i) = \xi_{jt} + \sigma_{jt}(i) + \epsilon^{(n)}_{jt}.
\]  

(1)

Consumers receive an additively-separable common component \( \xi_{jt} \), a type-specific shock \( \sigma_{jt}(i) \), and an idiosyncratic shock, \( \epsilon^{(n)}_{jt} \). The common component will typically be a function of firm \( j \)'s price, and takes the form \( \xi_{jt} = \tilde{\xi}_j + \alpha p_{jt} \) in the standard logit model (with \( \alpha < 0 \)). The type-specific shock, \( \sigma_{jt}(i) \), may be also be a function of firm \( j \)'s price, if consumers are less sensitive to the price of the product to which they are affiliated, as is the case in our empirical application.

We denote the probability that a consumer of type \( i \) chooses product \( j \) as \( s_{jt}(i) \). We normalize the utility of the outside good to be zero. Additionally, we normalize type-specific shocks of a type 0 consumer to be zero, i.e., \( \sigma_{jt}(0) = 0 \forall i \). Given the standard assumption of a type 1 extreme value distribution on the utility shock, \( \epsilon^{(n)}_{jt} \), the market shares of consumers conditional on type are:

\[
 s_{jt}(0) = \frac{\exp(\xi_{jt})}{1 + \sum_k \exp(\xi_{kt})}
\]  

(2)

\[
 s_{jt}(i) = \frac{\exp(\xi_{jt} + \sigma_{jt}(i))}{1 + \sum_{k/i} \exp(\xi_{kt} + \sigma_{jt}(i))}.
\]  

(3)

The observed share for firm \( j \) is given by the weighted average of choice probabilities by types:

\[
 S_{jt} = \sum_{i=0}^{I} r_{it} s_{jt}(i), \text{ where } r_{it} \text{ is the fraction of consumers of type } i.
\]

The mean utility \( \xi_{jt} \) may depend on time varying-observable characteristics as well as fixed effects. In the empirical application, we makes use of this latter feature to allow for serial correlation

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9Slade (1998) estimates a model of habit-forming consumers and sticky prices. That model, however, explicitly imposes a cost of price-adjustment. Our model does not rely upon a menu cost to explain dynamic price adjustments.
in unobservable utility shocks over time.

**Assumption 2: Single-Product Affiliation** We now place restrictions on the type-specific demand shocks, $\sigma_{jt}(i)$. We assume that each consumer type has an affiliation (utility shock) for a single product. Further, we assume that there is a single type corresponding to each product. Thus, a consumer of type $j$ is affiliated to product $j$. The type-specific demand shocks for other products are zero, i.e. $\sigma_{jt}(i) = 0 \forall i \neq j$.

This model has a direct interpretation of switching cost (or brand loyalty) when the cost (or benefit) is uniform relative to the other products. In our habit-formation approach, we allow the perceived net benefit to depend on the price of the good for the matched type $j$ as well as time-varying product characteristics. We say that a consumer of type $j$ is affiliated with product $j$, as this consumer has a perceived benefit of $\sigma_{jt}(j)$ relative to unaffiliated (type 0) consumers.

**Assumption 3: Evolution of Consumer Types** Types are not fixed for a consumer, but they may depend dynamically on previous behavior. We assume that the evolution of types follows a Markov process, where the state can be expressed as a function of the joint distribution of types and choices in the previous period.

Given the previous assumption that there is a one-to-one mapping between types and an affiliation for each product, we implicitly assume that consumers are symmetric within a market. Thus, we can express the distribution of consumer types in any period as a function of the distribution of choices in the previous period. For example, the share of consumer type $j$, $r_{jt}$, might be expressed as $r_{jt} = f(S_{jt(t-1)})$, where $S_{jt(t-1)}$ is the aggregate share of consumers (across all types) that chose product $j$ in the previous period.

We make the assumption that consumers that purchase product $j$ become affiliated with product $j$ in the next period at a given rate: $r_{jt} = \delta_{jt}S_{jt(t-1)}$. Under our habit-formation interpretation, a (random) fraction $\delta_{jt}$ of consumers that purchased $j$ in the previous period are habituated to product $j$. We allow this fraction to vary with observable characteristics of the firm and the market. The remaining $(1 - \delta_{jt})S_{jt(t-1)}$ consumers transition to type 0, along with any consumers that chose the outside good.

### 2.2 Supply

We assume that firms set prices to maximize the net present value of profits. In contrast to the typical setup for a dynamic game, we make relatively weak assumptions about the perceived continuation value, which depends on expectations and discount rates. We restrict our attention to Markov perfect equilibria.

First, we define firm $j$’s aggregate share as:

$$S_{jt} = (1 - \sum_{i \neq 0} r_{it})S_{jt(0)} + \sum_{i \neq 0} r_{it}S_{jt(i)}. \quad (4)$$

Thus, a firm’s total share of sales can be written as a weighted sum of its share of unaffiliated
consumers, $s_{jt}(0)$, and affiliated consumers, $s_{jt}(i)$. Note that firm $j$ will make sales to consumers affiliated to other firms $j \neq i$, but the probability that such consumers will choose firm $j$ is strictly lower than the choice probability of an unaffiliated consumer when the utility shocks $\{\sigma_{jt}(i)\}$ are positive.

**Assumption 4: Competition in Prices** We assume that firms set prices in each period to maximize the net present value of profits from an infinite-period game. Prices are set as a best response conditional on the state and contemporaneous prices of rival products. Firms cannot commit to future prices. The state vector in each period is summarized by marginal costs, $c_t$, the distribution of consumer types, $r_t$, and other variables that are captured by the vector, $x_t$, such as expectations about future costs. Entry is exogenous.\(^{10}\) The firm’s objective function can be summarized by the Bellman equation:

$$V_j(c_t, r_t, x_t) = \max_{p_{jt}|p_{kt}, k \neq j} \left\{ (p_{jt} - c_{jt})S_{jt} + \beta E(V_j(c_{t+1}, r_{t+1}, x_{t+1})|p_t, c_t, r_t, x_t) \right\}. \tag{5}$$

Prices in each period optimize the sum of the continuation value and current-period profits, $(p_{jt} - c_{jt})S_{jt}$. Both of these components depend upon marginal costs and the distribution of consumer types, $r_t$. Thus, firms anticipate both future marginal costs and how price affects the future distribution of consumer types. Note that the state space does not include previous period prices. We therefore exclude strategies that depend directly upon competitors’ historical prices, such as many forms of collusion.

**Assumption 5: Expectations** We place minimal restrictions on the expectations and discount factors for each firm. Consistent with the Markov perfect framework, we make the relatively weak assumption that the continuation value function is stable conditional on the state and prices.

Thus, market equilibrium is characterized by consumers making (myopic) utility-maximizing purchase decisions and firms pricing as the best response to other firm’s prices, conditional on the state.

### 2.3 Theoretical and Numerical Analysis

To develop intuition about pricing incentives when consumers are habit forming (or affiliated), we consider a deterministic setting where marginal costs are constant. Assuming Bertrand price-setting behavior results in a unique and stable steady-state, which is the equilibrium we focus on. First, we consider a monopolist. Second, we analyze a duopoly setting to examine post-merger price incentives.

#### 2.3.1 Monopoly

We analyze steady-state prices in a monopoly market (with an outside good) to show how habit-forming consumers affect optimal prices and markups.

\(^{10}\)Entry and exit are infrequent in retail gasoline.
To simplify notation in the monopoly case, let the monopolist’s share of affiliated and unaffiliated consumers be $s_j$ and $s_0$, respectively, and its number of affiliated consumers be $r$. Consumers become affiliated at rate $\delta$. We assume positive dependence in purchase behavior, so that $s_j > s_0$. In the steady-state, $r_{jt} = r_{jt+1} = r_j$ and $c_{jt} = c_{jt+1} = c_j$. The steady-state number of affiliated consumers, $r^{ss}$, is:

$$r = \delta((1-r)s_0 + r \cdot s_j)$$

$$r^{ss} = \frac{\delta s_0}{1 - \delta(s_j - s_0)}.$$ 

The steady-state number of affiliated consumers is increasing in the probability of becoming affiliated, $\delta$, and the difference between the choice probabilities of affiliated consumers and unaffiliated consumers, $s_j - s_0$. Using the steady-state value of affiliated consumers, we can solve for the steady-state pricing function.

The steady-state period value is:

$$V^{ss}(r^{ss}, c^{ss}) = (p^{ss} - c^{ss})((1 - r^{ss})s_0 + r^{ss} s_j) + \beta V^{ss}$$

$$= \frac{p^{ss} - c^{ss}}{1 - \beta} \cdot \frac{s_0}{1 - \delta(s_j - s_0)}.$$ 

This equation represents the monopolists discounted profits, conditional on costs remaining at its current level. Thus, profits are increasing in both $\delta$ and the difference in choice probabilities of affiliated and unaffiliated consumers. These results are straightforward: affiliated consumers are profitable. Also, note that a model with no affiliation is embedded in this formulation ($\delta = 0$ and $s_j = s_0$), in which case profits are simply the per-unit discounted profits multiplied by the firm’s market share.

Maximizing the steady-state value with respect to $p^{ss}$ yields the firm’s optimal pricing function:

$$p^{ss} = c^{ss} + \frac{-s_0 \left(1 - \delta s_j + \delta s_0\right)}{\frac{dx}{dp} \left(1 - \delta s_j\right) + \frac{dx}{dp} \delta s_0}.$$ 

$m = \text{markup of price over marginal cost}$

(6)

The second term, $m$, on the right-hand side of equation (6) captures the extent to which the firm prices above marginal cost (in equilibrium). As this markup term depends upon choice probabilities, it is implicitly a function of price. Thus, as in the standard logit model, we cannot derive an analytical solution for the steady-state price. Nonetheless, we derive a condition below to see how markups are impacted by consumer affiliation. In the usual case, $m$ will be declining in $p$, ensuring a unique equilibrium in prices.

Are markups higher or lower in the presence of habit formation? When habit formation is absent, $\delta = 0$ and $s_j = s_0$, equation (6) reduces to the first-order condition of the static model, $p^{ss} = c^{ss} - \frac{s_0}{dx_0/dp}$. Denoting the markup term with habit formation as $m_d$ and the markup term from the
static model as $m_s$, we compare these two terms at the solution to the static model:

$$m_d = - \frac{s_0 (1 - \delta s_j + \delta s_0)}{\frac{ds_0}{dp} (1 - \delta s_j) + \frac{ds}{dp} \delta s_0} \geq - \frac{s_0}{ds_0/dp} = m_s.$$ 

For a given price, the terms $s_0$ and $ds_0/dp$ are equivalent across the two models. Rearranging terms, we obtain a simple condition relating the levels of the markup terms:

$$m_d > m_s \iff - \frac{\partial s_0}{\partial p} > - \frac{\partial s_j}{\partial p}.$$  

(7)

A higher value for $m_d$ indicates higher markups and higher prices. Thus, if affiliated consumer quantities are relatively less sensitive to changes in price, then markups are higher.

This is an intuitive result. However, there is a nuanced point to this analysis, arising from the fact that there is not a direct mapping between our assumption of positive dependence and the condition in (7). Given our extension of the logit formulation, $\frac{\partial s_0}{\partial p} = \frac{\partial \xi}{\partial p} s_0 (1 - s_0)$ and $\frac{\partial s_j}{\partial p} = (\frac{\partial \xi}{\partial p} + \frac{\partial \sigma}{\partial p}) s_j (1 - s_j)$. Thus, whether or not markups are higher depends on the derivative of the type-specific shock with respect to price and the relative distance of $s_0$ and $s_j$ from 0.5 (at which point $s(1 - s)$ is maximized). Therefore, steady-state markups may be higher or lower with the presence of consumer affiliation. If we make the additional assumption that affiliated consumer utility is less sensitive to price, i.e., $- \frac{\partial \xi}{\partial p} > - (\frac{\partial \xi}{\partial p} + \frac{\partial \sigma}{\partial p})$, we might expect that markups are higher in the presence of consumer affiliation. However, the results show that it is still ambiguous whether markups are higher in the steady state, as $s_j$ may be close enough to 0.5 relative to $s_0$ to flip the inequality.

Thus, the presence of positively affiliated consumers may, counter-intuitively, lower the steady-state price, relative to the static model. The intuition for this result is akin to those summarized in Farrell and Klemperer (2007); with dynamic demand and affiliation, firms face a trade-off between pricing aggressively today and “harvesting” affiliated consumers in future periods. In the steady state, our model shows that either effect may dominate.

This finding has important implications for counterfactual exercises, such as merger simulation. Failing to account for consumer affiliation will result in inferring markups from incorrect first-order conditions, which is a critical input to merger simulation. Furthermore, antitrust agencies often infer elasticities from markups calculated using accounting data (see, Miller et al. (2013)); our result demonstrates that this will lead to incorrect elasticities and merger price predictions when affiliation (arising from habit formation, brand loyalty, or switching costs) is important.

### 2.3.2 Consumer Affiliation and Mergers

We now introduce competition into the model, and use numerical methods to analyze the steady-state of a duopoly market. In doing so, we demonstrate how the magnitude of the affiliation rate, $\delta$, affects prices, markups, and profits. Furthermore, we analyze the impact of a merger to monopoly and find that the percentage change in price is not monotonic in $\delta$ and may even decrease with $\delta$. We also analyze the consequences of ignoring the affiliation effect when calibrating demand and performing a merger simulation, and find that assuming the standard logit model when affiliation is present causes a systematic overestimation of the true price effect of the merger.
We specify the utility of an affiliated consumer as follows,

\[ u_{jt}^{(n)}(i) = \xi_j + \alpha p_{jt} + \mathbb{1}[i = j](\xi + \pi p_{jt}) + \epsilon_{jt}^{(n)}. \]  

(8)

Here, \( \mathbb{1}[i = j] \) takes a value of one if a consumer of type \( i \) purchases from firm \( j = i \), and zero otherwise. For unaffiliated consumers, the indicator function equals zero for all \( j \). This specification reduces to the standard logit model if \( \xi = \pi = 0 \).

Each firm is specified to sell a single product and maximize the expected discounted value of profits. Therefore, firm 1’s Bellman equation is as follows:

\[ V_1(r_1, r_2) = \max_{p_1, p_2} \left\{ (p_1 - c_1)((1 - r_1 - r_2)s_1(0) + r_1s_1(1) + r_2s_1(2)) + \beta(V_1(r_1', r_2')) \right\}. \]  

(9)

Here we drop the expectations operator, as the only source of uncertainty in the model is the realizations of marginal costs, which are fixed in the steady-state. To find the steady-state prices and affiliated shares for each firm, we focus on Markov perfect equilibrium.\(^{11}\) Firm 1’s profit-maximizing first-order condition is then:

\[ \frac{d\pi_1}{dp_1} + \beta \left( dV_1' \frac{d r_1'}{dp_1} + dV_1' \frac{d r_2'}{dp_2} \right) = 0. \]  

(10)

Firm 2’s first-order condition is defined analogously. Next, we specify the derivatives of equation (9) with respect to \( r_1 \) and \( r_2 \) and evaluate them at the prices that solve each firm’s first-order condition, which will be the prevailing prices at the steady-state. These two conditions are:

\[ \frac{dV_1}{dr_1} = \frac{\frac{d\pi_1}{dr_1} + \beta \left( dV_1' \frac{d r_1'}{dr_1} + dV_1' \frac{d r_2'}{dr_2} \right)}{1 - \beta \left( \frac{dr_1'}{dr_1} + \frac{dr_2'}{dr_2} \right)} \]  

(11)

\[ \frac{dV_1}{dr_2} = \frac{\frac{d\pi_1}{dr_2} + \beta \left( dV_1' \frac{d r_1'}{dr_2} + dV_1' \frac{d r_2'}{dr_2} \right)}{1 - \beta \left( \frac{dr_1'}{dr_2} + \frac{dr_2'}{dr_2} \right)}. \]  

(12)

In the steady-state, \( \frac{dV_1'}{dr_1} = \frac{dV_1'}{dr_2} = \frac{dV_1'}{dr_2} \) and \( \frac{dV_1'}{dr_2} = \frac{dV_1'}{dr_2} \). Therefore, we can plug equation (12) into equations (10) and (11) to eliminate \( \frac{dV_1'}{dr_1} \) and then equation (11) into equation (10) to eliminate \( \frac{dV_1'}{dr_2} \). This yields a steady-state profit-maximizing condition for firm 1 and the analogous one can be derived for firm 2. Then, in the steady-state, \( r' = r \), and therefore the equations that govern the evolution of affiliated consumers, \( r'_j = \delta S_j \), can be leveraged to yield two more equilibrium restrictions:

\[ r_1 = \frac{s_1(0) + r_2(s_1(2) - s_1(0))}{\frac{1}{\beta} + s_1(0) - s_1(1)} \]  

(13)

\[ r_2 = \frac{s_2(0) + r_1(s_2(1) - s_2(0))}{\frac{1}{\beta} + s_2(0) - s_2(2)}. \]  

(14)

\(^{11}\)Although we do not prove that the equilibrium is unique, the simulation results support there being a single steady-state equilibrium.
Equations (13) and (14) in conjunction with the steady-state profit-maximization conditions yield four restrictions that facilitate solving for the equilibrium prices and affiliated shares. However, there are four additional unknowns embedded in the envelope conditions: \( \frac{dp_j}{dr_k} \) for \( j, k \in \{1, 2\} \). These values are determined by the model, and we solve for them numerically using a local approximation method. For details, see the Appendix.

Table 1: Duopoly Numerical Parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta )</td>
<td>Affiliation Rate</td>
<td>{0.05, 0.1, ..., 0.95}</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Price Coefficient</td>
<td>{-7, -6.9, ..., -4}</td>
</tr>
<tr>
<td>( \xi )</td>
<td>Price Affiliation Effect</td>
<td>3</td>
</tr>
<tr>
<td>( \xi_1, \xi_2 )</td>
<td>Intercepts</td>
<td>15, 15</td>
</tr>
<tr>
<td>( \bar{\xi} )</td>
<td>Intercept Affiliation Effect</td>
<td>2</td>
</tr>
<tr>
<td>( c_1, c_2 )</td>
<td>Marginal Costs</td>
<td>2, 2</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Discount Factor</td>
<td>0.96</td>
</tr>
</tbody>
</table>

Finally, to produce results we must parameterize the demand model and specify values for marginal costs. The simulation parameters are reported in Table 1. The steady-state pre- and post-merger prices for the first product are plotted in Figure 1.\(^{12}\) To simulate the merger, we assume all demand parameters and marginal costs remain the same and only the ownership structure changes. Prices are depicted for values of \( \delta \) ranging from 0.05 to 0.95 and \( \alpha = -6 \). We find that prices rise as the affiliation effect becomes stronger, both in the pre-merger duopoly and post-merger monopoly settings. Thus, in the steady-state, the “harvesting” effect dominates the incentive to invest in future demand. The pre-merger price for \( \delta = 0.95 \) is higher than the post-merger price for \( \delta = 0.05 \), indicating that affiliation may have a greater impact on price than a merger when no consumer affiliation is present.

Table 2 summarizes the numerical results. Given our chosen parameters, steady-state prices and margins monotonically increase with the probability of consumers becoming affiliated. As we showed in the previous section on the monopolist, this is not necessarily the case.\(^{13}\) For these parameters, competition for future affiliated consumers does not dominate the incentive to increase prices to a loyal consumer base. Also, prices and margins in Table 2 decrease as consumers become more price sensitive.

Interestingly, the percentage price increase from a merger is not monotonic in the probability of consumers becoming affiliated. When consumers are less elastic (\(|\alpha| = 4 \text{ or } 5\)) the percentage price effects are generally large, ranging from 15 percent to 40 percent. At these values of \(|\alpha|\), however, the percent price change from the merger decreases with a stronger affiliation effect. On the other hand, when consumers are relatively more price sensitive, merger price effects increase with the affiliation effect. The reason for these results is, in part, due to the relative value of competing for versus harvesting affiliated consumers. When \(|\alpha| \) is lower, holding \( \bar{\pi} \) constant, then affiliated consumers are less likely to choose the competing firm. Consequently, harvesting affiliated

\(^{12}\)The prices for both products are approximately equal.

\(^{13}\)Further, Dubé et al. (2009) demonstrate numerically that affiliation may lead to lower equilibrium prices in an oligopoly setting, and indeed does so in their empirical application.
Table 2: Duopoly Prices and Margins with Consumer Affiliation

<table>
<thead>
<tr>
<th>Affiliation Rate (δ)</th>
<th>0.1</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pre-Merger Price</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>α = -4</td>
<td>2.52</td>
<td>2.58</td>
<td>2.71</td>
<td>3.00</td>
<td>3.31</td>
</tr>
<tr>
<td>α = -5</td>
<td>2.41</td>
<td>2.45</td>
<td>2.54</td>
<td>2.69</td>
<td>2.84</td>
</tr>
<tr>
<td>α = -6</td>
<td>2.31</td>
<td>2.33</td>
<td>2.38</td>
<td>2.45</td>
<td>2.53</td>
</tr>
<tr>
<td>α = -7</td>
<td>2.21</td>
<td>2.22</td>
<td>2.24</td>
<td>2.28</td>
<td>2.34</td>
</tr>
<tr>
<td><strong>Post-Merger Price</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>α = -4</td>
<td>3.54</td>
<td>3.58</td>
<td>3.66</td>
<td>3.82</td>
<td>4.02</td>
</tr>
<tr>
<td>α = -5</td>
<td>2.91</td>
<td>2.94</td>
<td>3.00</td>
<td>3.12</td>
<td>3.28</td>
</tr>
<tr>
<td>α = -6</td>
<td>2.51</td>
<td>2.53</td>
<td>2.58</td>
<td>2.67</td>
<td>2.79</td>
</tr>
<tr>
<td>α = -7</td>
<td>2.27</td>
<td>2.29</td>
<td>2.32</td>
<td>2.38</td>
<td>2.47</td>
</tr>
<tr>
<td><strong>%Δ Price from Merger</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>α = -4</td>
<td>40.1</td>
<td>38.8</td>
<td>35.1</td>
<td>27.2</td>
<td>21.4</td>
</tr>
<tr>
<td>α = -5</td>
<td>20.6</td>
<td>19.9</td>
<td>18.2</td>
<td>15.7</td>
<td>15.4</td>
</tr>
<tr>
<td>α = -6</td>
<td>8.9</td>
<td>8.8</td>
<td>8.6</td>
<td>9.0</td>
<td>10.4</td>
</tr>
<tr>
<td>α = -7</td>
<td>3.0</td>
<td>3.2</td>
<td>3.6</td>
<td>4.4</td>
<td>5.9</td>
</tr>
<tr>
<td><strong>Pre-Merger Margin</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>α = -4</td>
<td>0.21</td>
<td>0.22</td>
<td>0.26</td>
<td>0.33</td>
<td>0.40</td>
</tr>
<tr>
<td>α = -5</td>
<td>0.17</td>
<td>0.18</td>
<td>0.21</td>
<td>0.26</td>
<td>0.30</td>
</tr>
<tr>
<td>α = -6</td>
<td>0.13</td>
<td>0.14</td>
<td>0.16</td>
<td>0.18</td>
<td>0.21</td>
</tr>
<tr>
<td>α = -7</td>
<td>0.09</td>
<td>0.10</td>
<td>0.11</td>
<td>0.12</td>
<td>0.14</td>
</tr>
<tr>
<td><strong>Post-Merger Margin</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>α = -4</td>
<td>0.43</td>
<td>0.44</td>
<td>0.45</td>
<td>0.48</td>
<td>0.50</td>
</tr>
<tr>
<td>α = -5</td>
<td>0.31</td>
<td>0.32</td>
<td>0.33</td>
<td>0.36</td>
<td>0.39</td>
</tr>
<tr>
<td>α = -6</td>
<td>0.20</td>
<td>0.21</td>
<td>0.23</td>
<td>0.25</td>
<td>0.28</td>
</tr>
<tr>
<td>α = -7</td>
<td>0.12</td>
<td>0.12</td>
<td>0.14</td>
<td>0.16</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Notes: Statistics are for product 1. α is the price coefficient for unaffiliated consumers. The price coefficient for affiliated consumers is (α + π).
consumers is more valuable for a duopolist at lower levels of $|\alpha|$. In turn, pre-merger prices and competition is less intense, and post-merger monopolists have less consumer rent to extract (on a percentage basis). On the other hand, when $|\alpha|$ is high, then pre-merger competition for unaffiliated consumers is more intense and harvesting is relatively less valuable. Thus, increasing $\delta$ leads to relatively modest increases in pre-merger prices, and therefore the merger leads to greater price increases (on a percentage basis).

2.3.3 Model Misspecification in Merger Simulation

We now explore the implications of failing to account for consumer affiliation when calibrating demand and simulating a merger, as is commonly done antitrust practitioners. To do so, we consider the following hypothetical scenario. The true underlying model is the duopoly affiliation model. A practitioner observes both firms’ pre-merger prices, marginal costs, and aggregate market shares (rather than separately observing its unaffiliated and affiliated shares). This data is then used to recover the demand parameters of the standard logit model ($\xi_j$ and $\alpha$), and then the price effects of a merger are simulated.\textsuperscript{14}

Figures 2 (a) and (b) depict percent price increases from the true affiliation model (for product 1), and the predicted price increases from the standard logit model calibrated to match the observed pre-merger prices, marginal costs, and aggregate firm market shares. To be clear, both models are calibrated on the same data. In these figures, $\alpha$ is set to be -5.3 and -6.9, respectively. In both figures, the logit model over-predicts the price increase and the bias worsens as the affiliation effect increases. Thus, ignoring consumer affiliation becomes increasingly problematic as the effect strengthens.

These figures illustrate potential problems in antitrust enforcement if consumer affiliation is

\textsuperscript{14}See Miller et al. (2016) for details on the calibration and simulation procedure for the logit model.
not accounted for in merger simulation. Antitrust agencies typically evaluate mergers based upon whether or not a transaction will result in a “small but significant non-transitory increase in price,” which is often taken to be 5 percent.\textsuperscript{15} In Figure 2 (b), for values of $\delta$ from 0.55 and 0.75, the affiliation model predicts prices increases below a 5 percent SSNIP, whereas the incorrectly specified logit model finds a price change above the threshold. In Figure 2 (a), the price predictions of the two models diverge and become increasingly disparate as $\delta$ increases. While the price increases are above a SSNIP in both models, different demand parameters could result in similar price paths centered at a lower price increase, say 5 percent. Thus, depending on the underlying demand parameters, it could be that the affiliation and standard logit models lead to different enforcement decisions across nearly the entire range of $\delta$.

Table 17 in the Appendix provides simulation results for a broader range of parameters and confirms the patterns depicted in Figures 2 (a) and (b). The logit model calibrated to match pre-merger affiliation observables always over-predicts the true price effects. Interestingly, the price coefficient is always calibrated to be more elastic in the static model compared to the share-weighted affiliation price coefficient. Thus, biased predictions of the logit model arise from the omission of dynamic incentives to invest in future demand, rather than a biased elasticity or mean utility parameters alone. In the static logit model, there is no incentive for the post-merger monopolist to invest in future demand, and therefore it sets its post-merger price simply to balance current marginal revenue and marginal cost. On the other hand, in the dynamic model, there is still an incentive for the monopolist to invest in future demand, which imposes downward pressure on post-merger prices. Thus, starting from the same pre-merger prices the logit model leads to greater post-merger price effects, even when its demand parameters are biased toward greater elasticity and lower mean utility.

\textsuperscript{15}See the 2010 Horizontal Merger Guidelines issued jointly by the Federal Trade Commission and the US Department of Justice.
3 Reduced-Form Evidence of Dynamics

To motivate the structural model, we provide evidence of dynamic demand and dynamically adjusting retail gasoline prices. A host of previous studies have found that retail gasoline prices may take multiple weeks to fully incorporate a change in marginal cost.\(^\text{16}\) One innovation of our study is that we use separate measures of unexpected and expected costs to see if, consistent with forward-looking behavior, firms respond differentially to these two types of costs.

3.1 Data

The analysis relies upon daily, regular fuel retail prices for nearly every gas station in the states of Kentucky and Virginia, which totals almost six thousand stations. As a measure of marginal cost, the data include the brand-specific, daily wholesale rack price charged to each retailer. We therefore almost perfectly observe each gas station’s marginal cost changes, except for privately negotiated discounts per-gallon, which are likely fixed over the course of a year.\(^\text{17}\) The data ranges from September 25th, 2013 through September 30th, 2015. The data was obtained directly from the Oil Price Information Service (OPIS), which routinely supplies data used in academic studies (e.g. Lewis and Noel 2011; Chandra and Tappata 2011; Remer 2015).

OPIS also supplied the market share data, which is used by industry participants to track shares. It is calculated from “actual purchases that fleet drivers charge to their Wright Express Universal card.” The data is specified at the weekly, county/gasoline-brand level.\(^\text{18}\) Due to contractual limitations, OPIS only provided each brand’s share of sales, not the actual volume. Thus, to account for temporal changes in market-level demand, we supplement the share data with monthly, state-level consumption data from the Energy Information Administration (EIA).

3.2 Dynamic Demand: Correlation in Shares Over Time

Though ultimately the importance of demand-side dynamics in our data will be estimated by the model, it is informative to examine the reduced-form relationships between key elements. The dynamic model developed in the previous section is one in which today’s quantity depends on the quantity sold last period. As motivation for this model, we present the results from reduced-form regressions of shares on lagged shares in Table 3.

This exercise demonstrates that even after including rich fixed effects to capture static variation in consumer preferences, lagged shares are a significant predictor of current shares. The residual correlation in shares over time in our most detailed specification captures deviations from specific county-brand seasonal patterns. A positive correlation is consistent with state dependence in consumption. In specification (2), we show that lagged shares explain 95 percent of the variance in current shares, and the coefficient is close to one. In specification (3), we include measures of competition in the regressions, as well as a second-order polynomial in own price. The competition

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\(^\text{16}\)See Eckert (2013) for a comprehensive review of the literature.

\(^\text{17}\)The data also include all federal, state, and local taxes.

\(^\text{18}\)In some instances, the brand of gasoline may differ from the brand of the station. For example, some 7-Eleven stations in the data are identified as selling Exxon branded gasoline.
Table 3: Regressions with Share as the Dependent Variable

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>0.011***</td>
<td>0.000**</td>
<td>0.004</td>
<td>-0.002</td>
<td>-0.072***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.000)</td>
<td>(0.002)</td>
<td>(0.004)</td>
<td>(0.017)</td>
</tr>
<tr>
<td>Lagged Share</td>
<td>0.973***</td>
<td>0.963***</td>
<td>0.554***</td>
<td>0.628***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.002)</td>
<td>(0.003)</td>
<td></td>
</tr>
<tr>
<td>Price Squared</td>
<td>-0.000</td>
<td>0.001</td>
<td>0.010***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.001)</td>
<td>(0.003)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Comp. Price (Mean)</td>
<td>-0.004***</td>
<td>-0.006***</td>
<td>-0.106**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.044)</td>
<td></td>
<td></td>
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<tr>
<td>Comp. Price (SD)</td>
<td>-0.001</td>
<td>-0.002</td>
<td>0.088***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.024)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Comp. Stations</td>
<td>-0.000***</td>
<td>-0.000***</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0)</td>
<td></td>
<td></td>
</tr>
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<td>Num. Stations</td>
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<td>0.004***</td>
<td>0.004***</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Num. Brands</td>
<td>-0.001***</td>
<td>-0.002***</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Week FEs: X
County-Brand FEs: X
Brand-State-Week FEs: X
Week-County FEs: X
County-Brand-WofY FEs: X
Observations: 175566 170934 170774 170755 156210
$R^2$: 0.00 0.95 0.95 0.96 0.98

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

measures, which include the mean and standard deviations of competitor prices, are correlated with shares, but lagged shares still are the most important predictor of current shares. In specification (4), we include time and brand-county fixed effects. In the final specification, we include rich multi-level fixed effects: by county-brand-(week of year), brand-state-week, and week-county. The coefficient of 0.628 on lagged shares in this specification indicates that deviations in shares are highly correlated over time, even when we condition on the most salient variables that would appear in a static analysis, adjust for brand-county specific seasonal patterns, and allow for flexible brand-state and county time trends. This finding is consistent with demand-side dynamics, as there are patterns in shares over time that are challenging to explain with contemporaneous variables.19

19We have also estimated specifications that add lagged prices. Though the first lag is significant, there is almost no change to the lagged share coefficient.
3.3 Dynamic Pricing

We develop reduced-form results for evidence of dynamic pricing. Consistent with a model where firms accumulate affiliated consumers over time, we find that new entrants price lower relative to established competitors in the same market, and that this discount dissipates over time. Second, we examine cost pass-through and show that firms are slow to adjust to marginal cost changes. Moreover, firms anticipate expected changes in future costs by raising prices in advance of the change. The ability to separately estimate the response to expected and unexpected costs is a key innovation of our study. First, we detail how to decompose costs into an expected and an unexpected component. Then, we show pricing patterns pooled across all of the stations in our data. Finally, we analyze heterogeneity in station-level pricing behavior, and link the heterogeneity to measures of competition.

3.3.1 Dynamic Pricing of New Entrants

When forward-looking firms price to consumers that may become affiliated, there is an incentive to initially offer prices below the static optimum. In this setting, we expect a new entrant, all else equal, to initially price below its competitors. As the new entrant builds up its share of affiliated customers, its prices will gradually converge to its competition.

We test for and find evidence consistent with this dynamic pricing pattern in the data. To perform the analysis, we first identify a set of new entrants, defined as a gas station whose first price observation is at least six weeks after the start of the data and does not exit in the remainder of the sample. To ensure there is sufficient data and to control for composition effects in the analysis, we limit the set of entrants to those with at least one year of post-entry price data. Using this filter, we identify 193 entrants.
Figure 3 depicts the average difference between an entrant’s price and all other stations’ price in the same county, sorted by the number of weeks after entry. The figure demonstrates that gas stations enter with a price that is, on average, two cents per gallon less than incumbents’ prices. Entrants’ prices then slowly converge over time to the market average. A series of t-tests confirm the statistical significance of the results. For the first 8 weeks following entry, new entrants’ prices are significantly lower than the county average price.\textsuperscript{20} This pattern is consistent with a profit-maximizing firm building up an affiliated customer base over time, and raising its price to a gradually less elastic set of consumers.

\subsection{3.3.2 Cost Pass-through: Identifying Expected and Unexpected Costs}

We now analyze gas stations’ dynamic reactions to expected and unexpected costs. To disentangle the reaction to anticipated and unanticipated cost changes, we leverage data on wholesale gasoline futures traded on the New York Mercantile Stock Exchange (NYMEX). The presence of a futures market allows us to project expectations of future wholesale costs for the firms in our market.

To make these projections, we assume that firms are engaging in regression-like predictions of future wholesale costs, and we choose the 30-day ahead cost as our benchmark.\textsuperscript{21} Using station-specific wholesale costs, we regress the 30-day lead wholesale cost on the current wholesale cost and the 30-day ahead future. In particular, we estimate the following equation.

\begin{equation}
\begin{align*}
c_{it+30} = \alpha_1 c_{it} + \alpha_2 F_{t}^{30} + \gamma_i + \epsilon_{it}
\end{align*}
\end{equation}

Here, \(c_{it+30}\) is the 30-day-ahead wholesale cost for firm \(i\), \(F_{t}^{30}\) is the 30-day ahead forward contract price at date \(t\), and \(\gamma_i\) is a station fixed effect. We use the estimated parameters to construct expected 30-day ahead costs for all firms: \(\hat{c}_{it+30} = \hat{\alpha}_1 c_{it} + \hat{\alpha}_2 F_{t}^{30} + \hat{\gamma}_i\). The unexpected cost, or cost shock, is the residual: \(\tilde{c}_{it+30} = c_{it+30} - \hat{c}_{it+30}\).

For robustness, we construct a number of alternative estimates of expected costs, including a specification that makes use of all four available futures. However, we found that these alternative specifications were subject to overfit; the estimates performed substantially worse out-of-sample when we ran the regression on a subset of the data. Our chosen specification is remarkably stable, with a mean absolute difference of one percent when we use only the first half of the panel to estimate the model. Expected costs constitute 74.6 percent of the variation in costs (\(R^2\)) in our two-year sample, which includes a large decline in wholesale costs due to several supply shocks in 2014.

\textbf{A Note on 30-Day Ahead Expectations}

One of the challenges in discussing expectations is that they change each day with new information. News about a cost shock 30 days from now may arrive anytime within the next 30 days, if it has not arrived already. Therefore, any discussion of an “unexpected” cost shock must always be qualified...
with an “as of when.” Given previous findings in the gasoline literature indicating that prices take approximately four weeks to adjust, a 30-day ahead window seems an appropriate one to capture most of any anticipatory pricing behavior. Additionally, our findings support this window as being reasonable in this context. We see no relationship between unexpected costs or expected costs and the price 30 days prior.\(^\text{22}\)

### 3.3.3 Pass-through Regressions

Further highlighting the temporal component of cost pass-through, we separately estimate how gas stations react to expected versus unexpected cost changes. Beyond motivating the structural model, these results also demonstrate the importance of capturing firms’ anticipated price responses when estimating cost pass-through rates. For example, to analyze how much of a tax increase firms will pass-on to consumers, it is imperative to recognize that firms may begin to adjust their prices prior to the tax increase being enacted; failure to account for this response may lead to underestimating pass-through rates.

We employ our measures of expected and unexpected costs to study how firms differentially respond to these costs. We incorporate the main components of marginal costs for retail gasoline, which include the wholesale cost of gasoline and the per-unit sales tax. We estimate the following model:

\[
p_{it} = \sum_{s=-50}^{50} \beta_s \hat{c}_{it-s} + \sum_{s=-50}^{50} \gamma_s \tilde{c}_{it-s} + \sum_{s=-50}^{50} \phi_s \tau_{it-s} + \psi_i + \varepsilon_{it}. \tag{16}
\]

Here, \(p_{it}\) is the price observed at gas station \(i\) at time \(t\). \(\hat{c}_{it-s}\) and \(\tilde{c}_{it-s}\) are the expected and unexpected wholesale costs observed with lag \(s\), and \(\tau_{it-s}\) is the state-level sales tax.\(^\text{23}\) Using the estimated coefficients on the cost measures, we construct cumulative response functions to track the path of price adjustment to a one time, one unit cost change at time \(t = 0\). We incorporate 50 leads and lags to capture the full range of the dynamic response. We focus our results on unexpected and expected costs, as we do not have enough tax changes in our data to estimate a consistent pattern of response.\(^\text{24}\)

Figure 4 plots the cumulative response functions for unexpected and expected costs. Panel (a) displays the results for unexpected costs. Prices react suddenly and quickly at time zero, but it takes about four weeks for the prices to reach the new long-run equilibrium, reaching a peak of 0.71 after 34 days.

Panel (b) displays the cumulative response function for expected costs. Notably, firms begin to react to expected costs approximately 28 days in advance, with a relatively constant adjustment rate until the new long-run equilibrium is reached 21 days after the shock. Though the total duration of adjustment is longer compared to the unexpected cost shock, the firm incorporates the cost more

\(^{22}\)We interpret slight deviations from a zero as arising from an underlying correlation in unobserved cost shocks.

\(^{23}\)To more easily incorporate future anticipated costs into the regression, we do not estimate an error-correction model (Engle and Granger, 1987), which is commonly used to estimate pass-through in the retail gasoline literature.

\(^{24}\)As a robustness check, we also estimated the price response to expected and unexpected costs using the error-correction model, and we found nearly identical results.
quickly after it is realized. This coincides with substantial anticipation by the firm; the price already captures about a third of the effect of the expected cost shock the day before it arrives.

A striking result from these estimates is the difference in the long-run pass-through rates. Expected costs experience approximately “full” pass-through - a cost increase leads to a corresponding price increase of equal magnitude. On the other hand, unexpected costs demonstrate incomplete pass-through, moving about only 66 cents for each dollar increase in cost.

The different response to unexpected and expected costs emphasizes the need for empirical researchers to think carefully about designing proper estimators for pass-through. If costs are anticipated, then a pass-through measure that omits leads will only capture a portion of the overall response. Additionally, if a pass-through estimate is to be used for evaluation, it is important to ensure that the estimator relies on a mix of unexpected and expected shocks that translate to the policy under analysis. In our setting, for example, an analysis that used an unexpected cost shock to predict the impact of a gasoline tax rate change would be inappropriate, as the tax change is anticipated and leads to a much greater price response.

An important consideration for pass-through when analyzing imperfect competition is the distinction between idiosyncratic costs and common costs. In the current section, we consider only the simple cut between unexpected and expected costs to focus attention on this previously unexplored dimension of pass-through. In Appendix B.2, we present results for common costs and idiosyncratic costs. In our setting, costs are highly correlated, with common costs tending to dominate idiosyncratic costs at moderate frequencies. Therefore, the results for common costs are very similar to those in this section. One distinction, however, is that pass-through of unexpected common costs is higher than the pass-through for total unexpected costs. For idiosyncratic costs, which we consider by controlling for the prices of rivals, pass-through is on the order of 0.04 to 0.06. This is not surprising for a highly competitive market such as retail gasoline.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>p10</th>
<th>p25</th>
<th>p50</th>
<th>p75</th>
<th>p90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anticipation</td>
<td>-24.5</td>
<td>-15.0</td>
<td>-4.1</td>
<td>-0.1</td>
<td>3.6</td>
</tr>
<tr>
<td>Rate</td>
<td>0.009</td>
<td>0.016</td>
<td>0.026</td>
<td>0.045</td>
<td>0.103</td>
</tr>
<tr>
<td>Duration</td>
<td>5.0</td>
<td>12.3</td>
<td>26.9</td>
<td>47.0</td>
<td>59.4</td>
</tr>
<tr>
<td>Finish</td>
<td>4.0</td>
<td>10.7</td>
<td>21.5</td>
<td>32.6</td>
<td>41.6</td>
</tr>
<tr>
<td>LR_PTR</td>
<td>0.278</td>
<td>0.458</td>
<td>0.657</td>
<td>0.862</td>
<td>1.146</td>
</tr>
<tr>
<td>Anticipation_Exp</td>
<td>-39.0</td>
<td>-33.0</td>
<td>-25.4</td>
<td>-14.0</td>
<td>-4.0</td>
</tr>
<tr>
<td>Rate_Exp</td>
<td>0.015</td>
<td>0.017</td>
<td>0.021</td>
<td>0.029</td>
<td>0.051</td>
</tr>
<tr>
<td>Duration_Exp</td>
<td>19.2</td>
<td>33.2</td>
<td>48.5</td>
<td>60.0</td>
<td>70.7</td>
</tr>
<tr>
<td>Finish_Exp</td>
<td>10.2</td>
<td>15.8</td>
<td>23.0</td>
<td>30.0</td>
<td>38.0</td>
</tr>
<tr>
<td>LR_PTR_Exp</td>
<td>0.860</td>
<td>0.940</td>
<td>1.015</td>
<td>1.100</td>
<td>1.209</td>
</tr>
</tbody>
</table>

**A Note on Asymmetry**

Several readers might wonder about the relevance of asymmetric pricing, i.e., whether the price response is the same for positive and negative cost shocks. In robustness checks, we find little evidence of asymmetry based on our empirical specification. We note that there are empirical reasons one might find asymmetry when the pricing function is curved and the empirical specification does not correct for the location on the pricing function, as we do in our specification.\(^{25}\)

### 3.3.4 Heterogeneity in Pricing Behavior

We now show that there is significant heterogeneity across firms in the dynamics of cost pass-through, and that it can, in part, be explained by the firm’s competitive environment. To do so, we estimate station-specific regressions of equation (16), and then construct a cumulative response function for expected and unexpected costs separately for each gas station. To summarize these response functions, we use non-linear least squares to fit a three-segment spline to each firm’s cumulative response function. We restrict the slopes of the first and last segments to equal zero. The middle segment captures the duration and magnitude of the price adjustment. This methodology allows us to estimate for each gas station (i) anticipation: how far in advance the firm begins responding to a cost shock, (ii) duration: how many days it takes to reach the new long-run equilibrium, and (iii) rate: what proportion of the cost change is passed through to price each day. We also construct the long-run pass-through rate (“LR_PTR”), which reflects the degree to which prices reflect costs at the end of the adjustment period.

Table 4 summarizes the estimated heterogeneity parameters. The five rows correspond to the response to unexpected costs, and the last five rows correspond to expected costs. These summary statistics align with the mean pass-through parameters estimated in the previous section. The median firm anticipate expected costs much earlier (25.4 days in advance) than unexpected costs (4.1 days in advance), and full pass-through is greater for expected costs (1.02) than unexpected costs.

\(^{25}\)For example, a concave pricing function would result in a finding of asymmetry, as, for any realized cost, the slope of the pricing function is greater if the cost has increased from the previous period compared to a price decrease.
costs (0.66). The median firm takes longer to incorporate expected costs, but tends to reach full pass-through around the same time for both types of shocks (21.5 versus 23.0 days after the shock).

Comparing the 10-90 percentile range of the estimates, we find that there is greater variation in the pricing response to unexpected costs. The median rate of adjustment is faster for unexpected costs (0.026/day compared to 0.012), but sees a spread of (0.009, 0.103) for the 10-90 percentile range, compared to (0.015, 0.051). Likewise, there is greater variation in the timing of when firms fully incorporate cost shocks in price and the full pass-through rates (0.278 to 1.146 for unexpected, 0.860 to 1.209). The long-run pass-through estimates for expected costs display a tendency toward homogeneity, as they are clustered around 1.

For the specification in this section, our fitted splines consist of three parameters, where we restrict the level of the first segment to be zero. In Appendix B.3, we consider another version that adds the level of the first segment as a fourth parameter. The results are consistent.

### 3.3.5 Does Competition Affect Pricing Behavior?

In this section, we project our estimated parameters onto a simple measure of market competition. In doing so, we demonstrate that competition has an important effect on the dynamics of cost pass-through, which further motivates the dynamic oligopoly model and demonstrates that firm expectations play an important role in determining cost pass-through rates. We perform a series of county-level regressions that relate the estimated parameters in Table 4 to the Herfindahl-Hirschman index (“HHI”) in the county.

To do so, we first calculate the median firm-level parameter in each county, and regress it on the HHI and the number of gas stations in the county. To calculate the HHI, we sum the square of each gasoline brand’s weekly county market share. We then take the average across all weeks for each county to use as an independent variable in the regression. We also include the number of gas stations in each county as a regressor. This accounts for variation in population and demand across counties in a simple way; without this control, counties with a large consumer base would, all else equal, have lower HHI.

The results of the series of regression are presented in Table 5, and demonstrate that competition affects pass-through dynamics. We find a significant relationship between the HHI and reaction to expected changes in cost. Specifically, the anticipated response (“Anticipation_exp”), the rate of adjustment (“Rate_exp”), and duration of adjustment (“Duration_exp”) are all significantly related to the county HHI such that less competition leads to later and quicker responses to anticipated cost changes. Interestingly, we find in this simple regression that the long-run pass through rate (“LR_PTR”) is unaffected by competition. Adding additional controls, however, such as population or mean income leads to positive and significant results, in some specifications. Thus, there is some evidence that less competition also leads to a greater proportion of expected cost changes to be passed on to consumers. On the other hand, we do not estimate a significant relationship between pass-through of unexpected cost shocks and the HHI. This underscores the importance of

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26 Results are qualitatively the same when using the mean parameter value. We choose the median, as the distribution of parameter estimates are slightly skewed and there are a few outliers.

27 We take the weekly average HHI, as we observe market share rather than quantity.
### Table 5: Pass-Through and County Competition

<table>
<thead>
<tr>
<th>Dep. Variable</th>
<th>LR PTR</th>
<th>LR PTR exp</th>
<th>Rate</th>
<th>Rate exp</th>
<th>Anticipation</th>
<th>Anticipation exp</th>
<th>Duration</th>
<th>Duration exp</th>
</tr>
</thead>
<tbody>
<tr>
<td>HHI</td>
<td>0.182</td>
<td>0.041</td>
<td>0.016</td>
<td>0.014**</td>
<td>−2.592</td>
<td>14.058**</td>
<td>−2.159</td>
<td>−16.221***</td>
</tr>
<tr>
<td></td>
<td>(0.138)</td>
<td>(0.055)</td>
<td>(0.023)</td>
<td>(0.007)</td>
<td>(4.051)</td>
<td>(5.662)</td>
<td>(7.212)</td>
<td>(8.312)</td>
</tr>
<tr>
<td>Number</td>
<td>0.000</td>
<td>−0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>−0.013</td>
<td>0.036</td>
<td>−0.002</td>
<td>−0.051</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.017)</td>
<td>(0.023)</td>
<td>(0.046)</td>
<td>(0.034)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.629***</td>
<td>1.014***</td>
<td>0.034***</td>
<td>0.021***</td>
<td>−5.493***</td>
<td>−28.502***</td>
<td>29.190***</td>
<td>53.559***</td>
</tr>
<tr>
<td></td>
<td>(0.044)</td>
<td>(0.019)</td>
<td>(0.008)</td>
<td>(0.003)</td>
<td>(1.415)</td>
<td>(1.848)</td>
<td>(2.744)</td>
<td>(2.758)</td>
</tr>
<tr>
<td>Observations</td>
<td>248</td>
<td>248</td>
<td>248</td>
<td>248</td>
<td>248</td>
<td>248</td>
<td>248</td>
<td>248</td>
</tr>
</tbody>
</table>

*Notes: Significance levels: * 10 percent, ** 5 percent, *** 1 percent. Robust standard errors in parentheses. Notes LR PTR is the long-run full pass-through rate. exp represents the reaction to an expected cost change, and other variables are reactions to unexpected cost changes. HHI is Herfindahl Hirschman index. Observations are at the county level. The dependent variable is the median estimated firm level parameter in each county. Robust standard errors in parentheses.*

Distinguishing between expected and unexpected cost changes when analyzing the interaction of competition and pass-through dynamics. In the following sections, we develop a structural model to capture these reduced-form pricing dynamics.

## 4 Empirical Application: Demand Estimation

We now present the empirical application of our model to the retail gasoline markets described in the previous section. First, we outline our estimation methodology. We divide it in two stages, as demand can be estimated independently of the supply-side assumptions. Our method of demand estimation relies on data that is widely used in static demand estimation: shares, prices, and an instrument. After outlining the methodology, we present results for demand estimation. In Section 5, we use the estimated demand system to analyze the dynamic incentives faced by suppliers. We use these results to consider a merger between two brands.

### 4.1 Demand Estimation Methodology

Given the dynamic extension of the logit demand system, we obtain the familiar expression for the log ratio of shares of unaffiliated consumers from Equation (2):

\[
\ln s_{jt}(0) - \ln s_{0t}(0) = \xi_{jt} \tag{17}
\]

Likewise, we obtain the following relation for the shares of affiliated consumers:

\[
\ln s_{jt}(i) - \ln s_{0t}(i) = \xi_{jt} + \sigma_{jt}(i) \tag{18}
\]

We can combine Equations (17) and (18) to obtain the following:

\[
\ln s_{jt}(i) - \ln s_{0t}(i) - (\ln s_{jt}(0) - \ln s_{0t}(0)) = \sigma_{jt}(i) \tag{19}
\]

Thus, we obtain a relationship between purchasing patterns that depends only on the affiliation.
shock. Empirically, we may parameterize $\sigma_{jt}(i)$ as a function of underlying “dynamic” parameters and the data.

### 4.1.1 Identification of Choice Distributions

A key challenge with aggregate data and unobserved heterogeneity is that we do not separately observe choice patterns by unobserved consumer type. In our context, we observe the aggregate share, $S_{jt}$, which is a weighted combination of the $\{s_{jt}(i)\}$ and depends on the distribution of affiliated consumers for each product $\{r_{jt}\}$. Observed shares are determined by the following:

$$S_{jt} = (1 - \sum_k r_{kt}) \cdot s_{jt}(0) + \sum_i r_{it} \cdot s_{jt}(i).$$

To separate out $s_{jt}(i)$ from $S_{jt}$, we leverage the structure of the model. With discrete types, we show exact identification of the choice distribution without supplemental assumptions.

**Proposition 1** With discrete types, the distribution of choice patterns is identified conditional on the distribution of types and type-specific shocks.

To show identification, we use the relations in the previous section to obtain the following expressions:

$$s_{jt}(0) = \left(\frac{s_{0t}(0)}{s_{0t}(j)} - 1\right) \frac{1}{\exp(\sigma_{jt}(j)) - 1}$$

$$s_{jt}(i) = s_{0t}(i) \frac{s_{jt}(0)}{s_{0t}(0)} \cdot \exp(\sigma_{jt}(i))$$

That is, the $J + J^2$ unknowns $\{s_{jt}(i)\}$, can be expressed in terms of the $J + 1$ unknowns $\{s_{0t}(j)\}$ and $s_{0t}(0)$.

The observed share equation gives us $J$ restrictions:

$$S_{jt} = (1 - \sum_k r_{kt}) \cdot \left(\frac{s_{0t}(0)}{s_{0t}(j)} - 1\right) \frac{1}{\exp(\sigma_{jt}(j)) - 1}$$

$$+ \frac{s_{jt}(0)}{s_{0t}(0)} \cdot \sum_i r_{it} \cdot s_{0t}(i) \exp(\sigma_{jt}(i))$$

And the final restriction, $1 - \sum_k s_{jt}(0) - s_{0t}(0) = 0$ identifies the shares $\{s_{jt}(i)\}$, conditional on the parameters $\theta$ governing $\{r_{jt}\}$ and $\{\sigma_{jt}(i)\}$, i.e., the unobserved heterogeneity parameters.

### 4.1.2 Computational Simplicity in Estimation

Though the distribution of unobserved choices is identified, solving for the pattern of choices in estimation is another matter. The traditional approach is to “concentrate out” the distribution of unobserved heterogeneity while using a contraction mapping to solve (implicitly) for the shares of
the type 0 consumers (as in Berry et al. (1995)). In our setting, our assumption of single-product affiliation allows us to reduce the computation burden, as the full distribution of choice patterns in each market can be calculated directly after solving a system of equations in two variables. Thus, we replace a contraction mapping over simulations with a (non-linear) equation solver to speed up estimation.

Above, we showed that the choice patterns can be expressed in terms of the \( J + 1 \) parameters \( \{ s_{0t}(j) \} \) in each market. We now show that the system reduces to two parameters in each market, where the remaining \( J - 1 \) parameters are solved for by a quadratic function.

Under the assumption of single-product affiliation, we obtain

\[
\sum_i r_{it} \cdot s_{0t}(i) \exp(\sigma_{jt}(i)) = \sum_i r_{it} s_{0t}(i) + (\exp(\sigma_{jt}(j)) - 1) r_{jt} s_{0t}(j).
\]

Then, we can write

\[
0 = \left[ \exp(\sigma_{jt}(j)) - 1 \right] \frac{r_{jt}}{s_{0t}(0)} s_{0t}(j)^2 + s_{0t}(j) \left( \exp(\sigma_{jt}(j)) - 1 \right) (S_{jt} - r_{jt}) + \frac{1}{s_{0t}(0)} \sum_{0,i} r_{it} s_{0t}(i) - \sum_{0,i} r_{it} s_{0t}(i)
\]

and solve for \( \{ s_{0t}(j) \} \) as a quadratic function of dynamic parameters, observables, and the two market-level parameters \( s_{0t}(0) \) and \( \sum_{0,k} r_{kt} s_{0t}(k) \).

### 4.1.3 Identification of the Demand Parameters

From above, we obtain the utility of the unaffiliated (type 0) consumer in each market, \( \{ \xi_{jmt} \} \). We now discuss identification of the demand parameters. First, we make the standard assumption that the utility is linear in characteristics:

\[
\xi_{jmt} = \alpha p_{jmt} + X_{jmt} \gamma + \eta_{jmt}.
\]

The utility depends on price, \( p \), which is endogenous, and exogenous covariates \( X \). The exogenous covariates may contain multi-level fixed effects. Using standard instrumental variable arguments, these linear parameters are identified, conditional on the non-linear parameters used to solve for \( \xi_{jmt} \).

As we have exact identification conditional on the non-linear parameters, we need to employ additional moments to achieve identification of the nonlinear parameters \( \theta \) that govern \( \delta_{jt} \) and \( \sigma_{jt}(j) \). Given the context of our model, where firms can predict serial correlation in demand shocks and, further, can control the demand shocks by altering prices, we impose the supplemental moments that any idiosyncratic product-time shocks that are not anticipated by the firm are uncorrelated over
time. We construct these residuals after allowing for time period fixed effects, brand fixed effects, and local seasonal patterns. That is, we assume $\text{Corr}(\eta_{jmt}, \eta_{jm}(t+1)) = 0$, where, again, $\{\eta_{jmt}\}$ are the residuals after accounting for multi-level fixed effects, including non-parametric time trends and product-specific fixed effects.

The residuals may contain real demand shocks or measurement error from our data. To identify $\theta$, we impose that these moments hold within each market, which provides us with sufficient moments to identify our parameters.\(^{28}\)

One interpretation of the imposition of these moments is that the model endogenizes systematic correlation in product-specific demand shocks over time, rather than treating such correlation a feature of the exogeneous stochastic process.

### 4.1.4 Implementation

To implement our estimator, we use a nested regression approach with the following steps:

1. First, pick values for the non-linear parameters $\theta$ that govern $\delta_{jt}$ and $\sigma_{jt}(j)$.
2. Calculate $r_{jt} = \delta_{jt}S_{jt} - 1$ for all periods except the first.
3. In each market, solve for $s_{0t}(0)$ and $\sum_{0,t} r_{it}s_{0t}(i)$ using the non-linear system of equations obtained previously. Find $s_{jt}(0)$ for each firm.
4. Run the regression implied by equation (17) using the $\{s_{jt}(0)\}$ obtained in the previous step to solve for the linear parameters $(\alpha, \gamma)$. Calculate the correlation of the residuals $\text{Corr}(\hat{\eta}_{jt}, \hat{\eta}_{j(t+1)})$ within each market.
5. Repeat 1-4 to find $\theta$ that minimizes the sum of squared correlations.

The regression for Equation (17) may involve instrumental variables and the use of panel data methods such as fixed effects. In our empirical application, we make use of both.

The estimation methodology employs two tricks to speed up the computation of the dynamic model. First, the explicit formula for $\{s_{jt}(0)\}$ means that the non-linear solver only has to find two parameters, $s_{0t}(0)$ and $\sum_{0,t} r_{it}s_{0t}(i)$, for each market-period. The quadratic form for the remaining unknowns results in fast calculation. Second, the linear form for the nested regression allows for a quick calculation of the inner part of the routine and allows for serial correlation in unobservables.

### 4.2 Data for Structural Model

We supplement our EIA-adjusted weekly brand-county share measures with the average prices for the brand in a week-county. To reduce the occurrence of zero shares, which do not arise in the logit model, we use a simple linear interpolation for gaps up to four weeks. For any gap greater than four weeks, we assume the station was not in the choice set for that gap. We drop any observations that

\(^{28}\) One could construct related moments by using lagged prices as instruments, under the assumption that the prices are uncorrelated with the innovation in the demand residual. Using the first three lags of prices produces similar empirical results.
Table 6: Summary Statistics by County

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Min</th>
<th>Pctl(25)</th>
<th>Pctl(75)</th>
<th>Max</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Num. Brands</td>
<td>4.42</td>
<td>1.51</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>8</td>
<td>252</td>
</tr>
<tr>
<td>Price</td>
<td>2.87</td>
<td>0.11</td>
<td>2.69</td>
<td>2.77</td>
<td>2.95</td>
<td>3.14</td>
<td>252</td>
</tr>
<tr>
<td>Wholesale Price</td>
<td>2.25</td>
<td>0.05</td>
<td>2.12</td>
<td>2.21</td>
<td>2.28</td>
<td>2.44</td>
<td>252</td>
</tr>
<tr>
<td>Margin</td>
<td>0.21</td>
<td>0.06</td>
<td>0.06</td>
<td>0.17</td>
<td>0.24</td>
<td>0.44</td>
<td>252</td>
</tr>
<tr>
<td>Num. Stations</td>
<td>22.65</td>
<td>28.52</td>
<td>1.12</td>
<td>7.00</td>
<td>25.48</td>
<td>243.47</td>
<td>252</td>
</tr>
</tbody>
</table>

Table 7: Retail Gasoline in Kentucky and Virginia: Oct 2013 - Sep 2015

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Min</th>
<th>Pctl(25)</th>
<th>Pctl(75)</th>
<th>Max</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share</td>
<td>0.141</td>
<td>0.110</td>
<td>0.0003</td>
<td>0.060</td>
<td>0.187</td>
<td>0.688</td>
<td>112,930</td>
</tr>
<tr>
<td>Price</td>
<td>2.870</td>
<td>0.529</td>
<td>1.715</td>
<td>2.383</td>
<td>3.310</td>
<td>4.085</td>
<td>112,930</td>
</tr>
<tr>
<td>Wholesale Price</td>
<td>2.256</td>
<td>0.527</td>
<td>1.245</td>
<td>1.754</td>
<td>2.673</td>
<td>3.545</td>
<td>112,930</td>
</tr>
<tr>
<td>Wholesale FE</td>
<td>2.260</td>
<td>0.031</td>
<td>2.205</td>
<td>2.229</td>
<td>2.291</td>
<td>2.365</td>
<td>112,930</td>
</tr>
<tr>
<td>Margin</td>
<td>0.206</td>
<td>0.115</td>
<td>-0.440</td>
<td>0.132</td>
<td>0.273</td>
<td>1.048</td>
<td>112,930</td>
</tr>
<tr>
<td>Num. Stations</td>
<td>5.256</td>
<td>7.165</td>
<td>1</td>
<td>2</td>
<td>6</td>
<td>83</td>
<td>112,930</td>
</tr>
<tr>
<td>Food</td>
<td>0.860</td>
<td>0.259</td>
<td>0.000</td>
<td>0.804</td>
<td>1.000</td>
<td>1.000</td>
<td>112,633</td>
</tr>
<tr>
<td>Supermarket</td>
<td>0.019</td>
<td>0.093</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>1.000</td>
<td>112,633</td>
</tr>
<tr>
<td>Car Service</td>
<td>0.145</td>
<td>0.261</td>
<td>0.000</td>
<td>0.000</td>
<td>0.200</td>
<td>1.000</td>
<td>112,633</td>
</tr>
<tr>
<td>Interstate</td>
<td>0.004</td>
<td>0.049</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>1.000</td>
<td>112,633</td>
</tr>
</tbody>
</table>

have missing prices, missing shares, or missing shares in the previous week. This includes dropping
the first week of data, for which we do not have previous shares.

To reduce the sensitivity of the analysis to brands with small shares and to make the counterfactual exercises more computationally tractable, we aggregate brands with small shares into a synthetic "fringe" brand. We designate a brand as part of the fringe if it does not appear in ten or more of the 252 markets (counties). Additionally, if a brand does not make up more than 2 percent of the average shares within a market, or 10 percent of the shares for the periods in which it is present, we also designate the brand as a fringe participant for that market. These steps reduce the number of observations from 194,275 down to 112,930. Additionally, this reduces the maximum number of brands we observe in a county to 8, down from 24. Thus, we analyze the pricing behavior of 16 brands, including the synthetic fringe.29

Table 6 provides summary statistics of the data for the 252 counties in KY and VA. There is variation in the number of brands we observe in each county, ranging from 1 to 8. There is cross-sectional variation in wholesale prices, margins, and the number of stations in each county.

Table 7 provides summary statistics for the observation-level data in the analysis. The greatest number of stations a brand has in a single county in our data is 83. The 25th percentile is 2, and we have several observations of a brand with only a single station in our market. The variable

29Summary statistics by brand are presented in Table 19 in the Appendix. The fringe brand is, on average, 14 percent of the shares for the markets that it appears in. As we designate a fringe participant in nearly every market, the aggregated fringe has the highest overall share (12 percent).

28
Wholesale FE is the average wholesale price for a brand within a county. We interact this variable with the U.S. oil production data to generate an instrument for price in the demand estimation. For the 297 observations that are missing station-specific amenities, we impute the values as the market-period mean.

We also take steps to reduce measurement error in the number of stations in our data. We assume that stations exist for any gaps in our station-specific data lasting less than 12 weeks. Likewise, we trim for entry and exit by looking for 8 consecutive weeks (or more) of no data at the beginning or end of our sample.

### 4.3 Results: Demand Estimation

For the empirical application, we implement the methodology described in Section (4.1). Conditional on dynamic parameters, we extract the unobserved shares for all unaffiliated type consumers. We then estimate demand using the typical logit demand regression. Our chosen dynamic parameters minimize the average correlation in brand-market shocks over time (contemporaneous with a single-period lag), where the correlation is calculated within each market.\(^{30}\)

Our regression equation takes the following form:

\[
\ln \left( \frac{s_{jmt}(0)}{s_{0mt}(0)} \right) = \alpha p_{jmt} + \gamma_1 N_{jmt} + X_{jmt}\gamma_2 + \phi_t + \zeta_{jm} + \psi_{m,q(t)} + \eta_{jmt}
\]

Here, the subscript \(m\) denotes the market (county). We have shares and prices at the brand-county-week level. Within-county shares of unaffiliated consumers depend on prices, the number of stations for the brand in that market, \(N_{jmt}\), and brand-average station and demographic characteristics \(X_{jmt}\). The station amenities in \(X\) are indicators for the presence of food (snack or restaurant), co-location with a supermarket, car services, and proximity to an interstate. The (standardized) demographic characteristics are calculated at the census tract level and consist of median household income, population, population density, and commute percent. Each of the characteristics in \(X\) are weighted by the number of stations in each census tract within a county. Thus, (exogenous) entry and exit by a brand give us variation in these characteristics over time. We use brand-county fixed effects to capture heterogeneity in preferences, and we use the resulting correlation patterns over time to infer state dependence arising from consumer affiliation.

We allow for endogeneity in pricing behavior by instrumenting for \(p_{jmt}\) with predicted deviations in wholesale costs, where the predictions are obtained from a regression of deviations of wholesale costs (from the brand-state average) on the interaction of US production of crude oil (obtained from EIA) with the average wholesale cost for the brand in the state.\(^{31}\) This gives us brand-state-specific time variation in our instrument, and it is plausibly tied to variation in the wholesale cost and not linked to demand. We chose this measure, rather than instrumenting directly with brand-state wholesale costs, to allow for the possibility that local variation in wholesale costs over time may reflect brand-specific demand shocks.

\(^{30}\)In the estimated model, the implied correlation in shocks is 0.03.

\(^{31}\)Our measure of the average brand-state wholesale cost is the fixed effect obtained by a regression of wholesale costs on brand, state, and weekly fixed effects, thereby accounting for compositional differences in brand-states across time.
Figure 5: Shares and Prices

(a) Total Market Shares and Prices

(b) Instrument and Prices

Figure 5 summarizes the time-series variation by plotting mean total market shares and mean prices during our sample. In Panel (b), we show plot the mean instrument against the mean price. As the figure shows, there is a strong correlation with the instrument, constructed from US production of crude oil, and prices. Prices display seasonal patterns, reflecting demand, while our instrument does not.

In addition to the instrument, we employ panel data methods to address other unobservables. We allow for the fact that \( \xi_{jmt} \) may be correlated over time in ways not dependent on \((p,N,X)\). We let the unobserved components of demand be specified as \( \phi_t + \xi_{jm} + \psi_{m,q(t)} + \eta_{jmt} \). That is, we estimate weekly fixed effects, \( \{\phi_t\} \), and county-specific (quarterly) seasonal demand shocks, \( \{\psi_{m,q(t)}\} \), along with brand-county specific fixed effects, \( \{\xi_{jm}\} \). These fixed effects, which are at the most local level, account for cross-sectional heterogeneity in preferences.\(^{32}\)

Once we incorporate these fixed effects, the exclusion restriction for a valid use of our instrument is that the brand-market-period specific shock \( \eta_{jmt} \) is uncorrelated with the instrument, after accounting for aggregate period-specific shocks, county-level seasonal patterns, and brand-county level differences.\(^{33}\)

We allow the dynamic parameters \( \delta_{jmt} \) and \( \sigma_{jmt}(j) \) to depend upon time-varying station and demographic characteristics. We parameterize them as follows:

\[
\delta_{jmt} = \frac{\exp(\theta_1 + \theta_2 \cdot Income_{jmt})}{1 + \exp(\theta_1 + \theta_2 \cdot Income_{jmt})}
\]

\[
\sigma_{jmt}(j) = \theta_3 + \theta_4 \cdot Food_{jmt} + \theta_4 \cdot Fringe_{jmt} + \alpha \cdot p_{jmt}.
\]

Thus, we allow the rate of habituation to depend on the census-track weighted median household income for the brand in the market, and the strength of the habituation to depend on the presence

\(^{32}\)We benefit from the size of our dataset. All county-quarters have at least 25 observations, and 98.5 percent of county-brands have at least 40 observations.

\(^{33}\)As mentioned previously, one interpretation of our decomposition is that we attribute all of the brand-specific correlation in demand over time within a market to unobservable demand types arising from affiliation.
of food and whether or not the brand is a fringe brand. Additionally, we allow habituated consumers to become less (or more) price sensitive for the affiliated brand’s product. This change in price sensitivity is captured by $\alpha$. This specification allows for the possibility that income affects the propensity for consumers to form habits, and that the characteristics of the product affect the strength of the habit.

The estimates for the linear parameters are reported in Table 8. The first three columns report coefficient estimates using the observed shares in logit demand estimation. The fourth column reports the results from our dynamic model, along with the dynamic parameters.

The linear parameters specify demand for unaffiliated consumers. In the static model, all consumers are unaffiliated. As expected, we obtain a larger (in magnitude) price coefficient with our dynamic specification, as we allow for the presence of affiliated consumers with lower price sensitivities. The number of stations seems to matter less for attracting unaffiliated consumers after accounting for dynamics.

We note that the R-squared for the regression with unaffiliated consumers falls in the dynamic model. This is to be expected, as the transformation of the log ratio observed shares to unaffiliated shares is a non-affine transformation, and will not preserve the measure of R-squared. Specifically, the shares for unaffiliated consumers fall closer to zero, from 14 percent (observed) to 9 percent in the dynamic model, and the majority of the variance in aggregate shares is attributed to the variance in unaffiliated shares. This serve to reduce the R-squared under the log-ratio transformation.

Table 9 reports estimates of the dynamic parameters. The parameters $\theta_1$ and $\theta_2$ imply that 61
percent of consumers, on average, develop an affiliation for the brand they previously purchased from. The coefficient of 0.23 on income indicates that higher-income consumers are more likely to develop an affiliation, consistent with a habit-formation model where switching costs are increasing in wages. The next coefficient indicates that affiliated consumers are more likely to purchase from the previous brand, as expected. The strength of affiliation increases in the presence of food, and it is lower for fringe brands. Affiliated consumers have a low price sensitivity, as the net sensitivity \((\alpha + \pi)\) of -0.348 is quite small.

To interpret the price coefficients, we summarize the implied elasticities in Table 10. The affiliated consumers of our model are very inelastic, with a near-zero response to price effects. The unaffiliated consumers, however, are highly elastic, with an average own-price elasticity of -7.0. This is large in magnitude, and it implies that for a 1 percent increase in price (roughly 3 cents), the station will lose 7 percent of the unaffiliated consumers. This high level of price sensitivity for a subset of retail gasoline consumers seems plausible, as, anecdotally, some “shoppers” are known to go well out of the way to save a few cents per gallon.

The average weighted elasticity, which weighs affiliated and unaffiliated consumers by their relative (purchasing) proportions, is -2.6. This weighted elasticity is starkly different than the elasticity one obtains by estimating the static model. A “naive” estimate (supposing the true model were dynamic) of this elasticity would obtain a value of -6.2, which implies a much greater loss in market share for a given price increase than we obtain from the dynamic model. Indeed, the static model obtains an elasticity more in line with the unaffiliated elasticity than the overall elasticity from the dynamic model.

For ease of interpretation of the dynamic parameters, we present summary statistics and graphs
Table 11: Shares of Affiliated and Unaffiliated Consumers

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Min</th>
<th>Pctl(25)</th>
<th>Pctl(75)</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed Share ($S_{jmt}$)</td>
<td>0.141</td>
<td>0.110</td>
<td>0.000</td>
<td>0.060</td>
<td>0.187</td>
<td>0.688</td>
</tr>
<tr>
<td>Unaffiliated Share ($s_{jmt}(0)$)</td>
<td>0.088</td>
<td>0.078</td>
<td>0.000</td>
<td>0.034</td>
<td>0.119</td>
<td>0.980</td>
</tr>
<tr>
<td>Affiliated Share ($s_{jmt}(j)$)</td>
<td>0.990</td>
<td>0.058</td>
<td>0.052</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$r_{jmt}$</td>
<td>0.085</td>
<td>0.066</td>
<td>0.000</td>
<td>0.037</td>
<td>0.114</td>
<td>0.432</td>
</tr>
<tr>
<td>Portion from Unaffiliated</td>
<td>0.380</td>
<td>0.145</td>
<td>0.000</td>
<td>0.308</td>
<td>0.456</td>
<td>1.779</td>
</tr>
<tr>
<td>Portion from Affiliated</td>
<td>0.619</td>
<td>0.146</td>
<td>0.000</td>
<td>0.543</td>
<td>0.691</td>
<td>1.000</td>
</tr>
</tbody>
</table>

of the resulting shares. Table 11 displays the means and standard deviations for observed shares and the two components of the shares we identify. We also report the portion of the observed share that a firm realizes from the affiliated consumers, which is 62 percent on average. Firms (brands) retain nearly all of their affiliated consumers (99 percent), and capture 9 percent of the unaffiliated consumers on average.

In Figure 6, we display the time series of the average price and the average shares. The shares for the consumers that are unaffiliated, displayed in Panel (a), are negatively correlated with the price. Intuitively, consumers shift to the outside option when prices are higher. On the other hand, the share of affiliated consumers, displayed in Panel (b), track the prices more closely. This occurs because prices respond positively to demand shocks, as do the affiliated consumers, who we find to be price inelastic.

Finally, we show how the time series of shares differ over time between large brands and fringe brands. Panel (c) presents the unaffiliated shares over time. Large brands and fringe firms track each other closely. Panel (d) show the time series for affiliated consumers. Consumers show a greater strength of affiliation for a large brand than they do for fringe brands, as was captured by the dynamic parameter estimates. This finding is consistent with a model where brand loyalty affects the habit formation of consumers.

5 Empirical Application: Supply-Side Analysis

5.1 Supply-Side Estimation

Given the demand estimates, we construct the components in each firm’s Bellman equation from (5). Using the estimated demand parameters, we are able to recover the (derivative of) the continuation value. The dynamic condition for optimal pricing is:

$$\frac{\partial S_{jt}}{\partial p_{jt}} (p_{jt} - c_{jt}) + S_{jt} + \beta E \left[ V_j \left( r_{t+1}, c_{t+1}, x_{t+1} \right) \right]_{p_t, r_t, c_t, x_t} = 0$$

34 For the remainder of the figures in this section, including this one, our plots display equally-weighted four-week moving averages.

35 In Figure 9 and Figure 10 of the Appendix, we show how the elasticities change over time for unaffiliated and affiliated consumers, and also for large brands and fringe firms.
The estimation of dynamic parameters, along with our measures of marginal costs, allow for a direct estimate of the derivative of the static profit with respect to price: \( \frac{\partial S_{jt}}{\partial p_{jt}} (p_{jt} - c_{jt}) + S_{jt} \). If this were zero, it would imply that firms are pricing myopically in the context of the model, as they are simply maximizing the current-period profits.

When this is non-zero, we attribute the residual to the (negative) derivative of the continuation value, \( \beta E[V(c', r', x') | p, c, r, x] \), under the assumption of optimal price-setting behavior. Thus, our estimated derivative of the continuation value is that which rationalizes the pricing behavior of firms, conditional on the demand-side assumptions, the data, and Bertrand price competition.

We project the estimated residuals on prices and state variables, including measures that capture expectations. Under the Markovian assumptions, it is theoretically possible to write the continuation value as a function of the state variables and actions of the firms. We construct a reduced-form estimate of the derivative of this function. This allows us to perform a counterfactual analysis by accounting for how the dynamic incentives change with the state and the endogenous pricing decisions by firms. We use this estimated function to perform a counterfactual merger simulation.
Table 12: Summary of Implied $\beta \frac{\partial E[V_j(t)]}{\partial p_{jt}}$

<table>
<thead>
<tr>
<th>Group</th>
<th>Mean</th>
<th>Min</th>
<th>p25</th>
<th>Median</th>
<th>p75</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>-0.115</td>
<td>-0.749</td>
<td>-0.153</td>
<td>-0.087</td>
<td>-0.048</td>
<td>0.061</td>
</tr>
</tbody>
</table>

Notes: The table displays the estimated derivative of continuation value. A finding of zero would indicate the absence of forward-looking behavior by firms. Negative values indicate that firms are pricing lower in that period than the optimal myopic price.

5.2 Results: Supply-Side Behavior

Summary statistics for the value of the derivative of the continuation value (DCV) are presented in Table 12. The mean and median are negative, which means that firms typically price lower than the static profit-maximizing price implied by our demand model. The magnitudes are significant: the mean of -0.115 implies that a 1 cent increase in price would increase static profits by roughly 4 percent.\(^{36}\) We interpret this deviation from the static optimum as arising from a dynamic incentive, though other explanations may be plausible.\(^{37}\) Intuitively, firms are lowering prices to invest in future demand. This result, combined with our reduced-form findings of anticipatory pricing for expected costs, provides consistent evidence of forward-looking pricing behavior in retail gasoline.

We project the implied DCV onto costs, expectations of future costs, market structure variables, current affiliated consumers, and prices. The measure of affiliated consumers of the rivals and the rivals' mean price is constructed relative to respective variables of the firm. The results are reported in Table 13. The first specification reports the coefficients from a regression of the DCV onto the listed variables and all of the first-order interactions. The variables are de-meaned so that the interpretation of the reported coefficient is the marginal effect at the mean of the other variables. As the DCV is negative on average, a negative coefficient implies that the variable is associated with a stronger dynamic pricing incentive, or a greater deviation from the optimal static price.

To show more directly how sensitive firms are to dynamic considerations, the second column reports a regression where we replace the value of the DCV with the logged absolute value. Thus, the coefficients reflect the semi-elasticity for the magnitude of the dynamic incentive. Typically, a negative coefficient in the first column corresponds to a positive coefficient in the second.

After accounting for interactions among the variables, we find that higher marginal costs today increase the relative weight of dynamic incentives. This is consistent with a model where cost shocks may be temporary, and therefore the firm prefers to not raise the price to reflect the cost change immediately. We occasionally observe negative margins in the data, which would be consistent with this interpretation. Increases in expected future costs, however, reduce the magnitude of dynamic incentives. Intuitively, higher costs in the future leads firms to try to capture more profits today.

Firms that are in a less advantaged position, either through a more competitive market (more brands) or less affiliated consumers (outright, or relative to their rivals) seem to price closer to the static optimum. In other words, we estimate that less advantaged firms seem have less of a dynamic incentive to keep prices low, and more advantaged firms have a greater incentive to do so.

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\(^{36}\)The average (scaled) profit in our data is 0.029. Recall that margins are approximately 21 cents per gallon.

\(^{37}\)For example, a component of this residual may be profits obtained by complementary products, such as food sold at retail gasoline stations.
Table 13: Dynamic Pricing Incentive: Regressions

<table>
<thead>
<tr>
<th></th>
<th>Sensitivity</th>
<th>Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Marginal Cost</td>
<td>-0.123***</td>
<td>1.294***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.009)</td>
</tr>
<tr>
<td>Cost Change (30-Day Ahead)</td>
<td>0.011***</td>
<td>-0.042***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.010)</td>
</tr>
<tr>
<td>Num. Stations</td>
<td>-0.000</td>
<td>0.005***</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>Total Stations</td>
<td>0.000***</td>
<td>0.000***</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>Num. Brands</td>
<td>0.011***</td>
<td>-0.068***</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>Mean Affiliated (Rivals)</td>
<td>0.434***</td>
<td>-2.451***</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.111)</td>
</tr>
<tr>
<td>Mean Price (Rivals)</td>
<td>-0.001</td>
<td>-0.002</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.022)</td>
</tr>
<tr>
<td>Affiliated Customers</td>
<td>-0.769***</td>
<td>11.529***</td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
<td>(0.138)</td>
</tr>
<tr>
<td>Price</td>
<td>0.124***</td>
<td>-1.296***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.009)</td>
</tr>
<tr>
<td>First-Order Interactions</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>112,930</td>
<td>112,930</td>
</tr>
<tr>
<td>R²</td>
<td>0.950</td>
<td>0.862</td>
</tr>
</tbody>
</table>

Notes: Significance levels: * 10 percent, ** 5 percent, *** 1 percent. The table displays the estimated coefficients from a regression of the dynamic pricing incentive on state variables and the firm’s price. The regression includes first-order interactions of all of the displayed variables. The variables are de-meaned, so the coefficient is interpreted as the marginal association at the mean of the other variables. The second column reports the regression with a measure of sensitivity, which is the log absolute value of the dynamic pricing incentive. In general, a negative coefficient in the first column implies a greater sensitivity to dynamics when pricing, generating a positive coefficient in the second column.
5.3 Merger Simulation

To evaluate the effect of dynamic pricing incentives in a merger context, we simulate a merger between Marathon and BP, which are the number one and number four (non-fringe) brands in terms of overall shares in our sample. Out of the 252 markets, they overlap in 75. In these 75 markets, the average (post-merger) HHI is 1511, and the mean change in HHI resulting from the merger is 383. In 8 markets, the resulting HHIs are greater than 2500, and the changes are greater than 200, meeting the typical thresholds that are presumed likely to enhance market power. The merger would change twelve markets from 3 firms to 2 firms and eighteen markets from 4 firms to 3 firms. We allow the firms to merge at the beginning of September 2014, and we calculate counterfactual prices and shares for the second half of our sample.

To generate counterfactuals, we proceed under the assumptions that the demand model is an accurate approximation for capturing the static profit incentives, and that the residuals reflect dynamic pricing incentives. Under these assumptions, we construct a reduced-form approximation to the derivative of the continuation value to solve for equilibrium prices. In contrast to other approaches, such as estimation of the continuation value function through forward simulation, this greatly reduces computational time to re-compute the price equilibria and avoids the need to make dimension-reducing assumptions (such as constructing a limited grid for prices) that are less palatable in our setting. To accurately represent behavior, this approach requires that the state variables included in the reduced-form approximation capture the payoff-relevant states (including market structure) and also that the counterfactual states can be reasonably interpolated from the data.

For this approximation, we project the estimated derivative of the continuation value on state variables and prices. We use the regression from Table 13, which has an $R^2$ of 0.95, to approximate the change in the dynamic incentives in the post-merger environment. We note that this approximation is constructed using the entire sample of 252 markets and two years of data, which has a broader support in terms of state variables than the 75 markets analyzed in the counterfactual. The use of a broader sample to estimate the function, combined with the high degree of model fit, increases the likelihood that the counterfactual exercise is calculated roughly within the bounds of the estimation sample. Using this approximation for the dynamic pricing incentive, we re-compute the price-setting equilibrium in the first period after the merger and simulate forward.

In our merger counterfactual, we assume that consumer habits are tied to gas station locations and are unaffected by the brand name, and therefore the merging firms retain affiliated consumers from both Marathon and BP. Table 14 displays the mean effects of the merger. The effects are small, with an average price increase for the merging firms of two percent. Shares for these firms decline by 10 percent, and profits increase. Profits from the competitors fall, which results in a small net effect on industry profits. Interestingly, prices for competitors fall, which is the opposite of what we

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38As we treat the fringe as a profit-maximizing entity, we calculate HHI treating the fringe as one firm. This will overstate the baseline HHI and the price response by competitors. The change in HHI is unaffected by this abstraction.

39Here, again, we count the fringe brand as a single firm.

40We adjust the combined utility shocks so that the market shares would remain constant if the merged firms maintained the same (share-weighted) price.

41We recognize that out-of-sample prediction is a challenge for counterfactual analysis in any setting.

42For cost and demand shocks, we use the estimates from the data so that the comparison occurs along a particular realization of the stochastic path.
usually expect after a merger. Prices are strategic substitutes due to affiliation dynamics.

Our setting, with dynamic brand-specific effects, puts a greater emphasis on the what exactly a merger will be in practice. This is a nuanced question. Do the merging firms retain separate brands, or do they convert all stations to a single brand? Do affiliated consumers retain their affiliation to the merged company? We consider a second scenario where the acquiring firm loses the affiliated consumers from the acquired brand: all of the BP consumers become unaffiliated. Since the one-year post-merger window is sufficient to reach the new long-run equilibrium, the results look similar.\footnote{Prices for the merging firm increase by 2.5 percent and shares fall by 12 percent.}

In Table 15, we regress the percentage effects from our counterfactual first scenario, when affiliation transfers, on indicators of the strength of the merger: the HHI delta and the combined level of affiliated consumers for the merging brands. The coefficient on the first model indicates that an HHI change of 100 points would result, on average, in a 0.37 percent price change. For the second model, the coefficient of 11.4 indicates that a combined loyalty share of 0.1 would result in a 1.1 percent change in prices.

\begin{table}[h]
\centering
\caption{Merger Effects: Regressions}
\begin{tabular}{lcccc}
\hline
 & Price & Share & Profits \\
\hline
HHI Delta (100) & 0.372*** & -1.446*** & 2.100*** \\
 & (0.048) & (0.259) & (0.346) \\
Combined r & 11.379*** & -46.464*** & 60.850*** \\
 & (1.350) & (7.231) & (10.177) \\
Observations & 75 & 75 & 75 & 75 & 75 \\
R\textsuperscript{2} & 0.449 & 0.490 & 0.297 & 0.358 & 0.332 & 0.326 \\
\hline
\end{tabular}
\textit{Notes:} Significance levels: * 10 percent, ** 5 percent, *** 1 percent. The table displays regressions of the estimated merger effects for the merging brands on the change in HHI resulting from the merger and the combined locked-in customers entering the merger. No constant is used in the regressions.
\end{table}

Overall, these effects are small relative to what one might normally expect from a merger. Even for the 8 markets that fall under the category of likely to increase market power, the results are quite similar. For comparison, we report the results from a merger analysis using a static model in Table 16, which has average price effects of over 5 percent for the merging firms.\footnote{The static analysis uses the standard logit discrete choice demand system.} Notice that prices in the static model are strategic complements, as the prices for rivals rise. How do we interpret dynamic model? Though the merged firm has a static incentive to raise prices, the dynamic incentive...
is also strengthened, as the firm may accumulate more valuable dynamic assets. Thus, the dynamic incentives mitigate the incentive to raise prices post-merger, dampening the expected price effects.

### 6 Conclusion

We develop a model of dynamic demand that accounts for the slow adjustment of prices to changes in cost. The dynamics result from competing firms optimally setting prices to consumers that may become loyal or habituated to their current supplier. We show that firms pricing optimally to consumers with dynamic preferences may have higher or lower markups relative to static demand, depending upon whether the incentive to harvest affiliated consumers dominates the incentive to invest in future demand. In this setting, failing to account for dynamic demand will cause merger simulations to over-predict post-merger price increases.

Using data from retail gasoline markets, we first demonstrate that prices adjust slowly to cost changes, and that path of price adjustment depends upon whether the costs change is expected or unexpected. This finding demonstrates the importance of accounting for firm expectations when estimating pass-through and in the estimation of demand models that can accommodate dynamics.

We develop an empirical model that can identify dynamic demand parameters using data on price, shares, and an instrument. Results suggest that roughly 60 percent of retail gasoline consumers become affiliated to the firm from which they currently purchase on a week-to-week basis, and that these consumers are extremely price insensitive. Conversely, we find that unaffiliated consumers are quite price sensitive and play an important role in disciplining equilibrium prices. We evaluate the dynamic incentives affecting prices, and we show, both theoretically and empirically, that merger effects are muted by the presence of dynamic incentives.
References


A Duopoly Steady-State Analysis

We implement the following procedure to solve numerically for the duopoly steady state:

1. Provide an initial guess for $\frac{dp}{dr}$.
2. Solve for $p^{ss}$ and $r^{ss}$ using the steady-state restrictions.
3. Take the numerical derivative of $p^{ss}$ with respect to $r^{ss}$. We approximate the numerical derivative by slightly perturbing $r^{ss}$ by $h$, resolving for $p^{ss}$, and calculating $\frac{p^{ss}(r+h) - p^{ss}(r-h)}{2h}$.
4. Solve again for $p^{ss}$ and $r^{ss}$ using the steady-state restrictions and the updated $\frac{dp}{dr}$.
5. If the changes in $p^{ss}$ and $r^{ss}$ between steps 2 and 4 fall below a critical value, and the changes in each of the four elements of $\frac{dp}{dr}$ between steps 1 and 3 fall below a critical then we have found all steady-state values. If the change in any of the unknowns is above the critical value then repeat steps 1-4 using the updated values of $\frac{dp}{dr}$.

B Supplemental Tables and Figures

B.1 Numerical Merger Simulations

Table 17 provides simulation results for a broader range of parameters and confirms the patterns depicted in Figures 2 (a) and (b). The logit model calibrated to match pre-merger affiliation observables always over-predicts the true price effects. The magnitude of the bias (in terms of percentage points) increases with the affiliation effect and decreases with the magnitude of the unaffiliated price coefficient, $|\alpha|$. To calibrate the logit model, the price coefficient is inferred from observed margins. The table demonstrates the logit coefficient is always calibrated to be more elastic than the share-weighted affiliation price coefficient (columns 4 and 3, respectively). Also, while not depicted in the table, the mean utility in the logit model is also calibrated to be lower when compared to the share-weighted counterparts in the affiliation model. This occurs because, conditional on the observed margins, the logit model interprets the incentive to invest in future demand, which puts downward pressure on margins, as more elastic demand.

B.2 Pass-through for Common and Idiosyncratic Costs

In Figure 7, we present Pass-through results for common market-level costs. In Figure 8, we present Pass-through results controlling for the other firm's prices.
Table 17: Duopoly Merger Simulation: Lock-in vs. Logit

<table>
<thead>
<tr>
<th>δ</th>
<th>Free-Agent α</th>
<th>Lock-in Avg. α</th>
<th>Logit α</th>
<th>Pre $p_1$</th>
<th>Lock-in $Δp_1$</th>
<th>Logit $Δp_1$</th>
<th>Bias</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>-4</td>
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<td>-3.79</td>
<td>2.53</td>
<td>40.1</td>
<td>42.5</td>
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</tr>
<tr>
<td>0.25</td>
<td>-4</td>
<td>-3.25</td>
<td>-3.45</td>
<td>2.58</td>
<td>38.8</td>
<td>45.6</td>
<td>6.8</td>
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<tr>
<td>0.50</td>
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<td>-2.50</td>
<td>-2.79</td>
<td>2.71</td>
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<td>52.0</td>
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</tr>
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<td>27.2</td>
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<td>2.69</td>
<td>15.7</td>
<td>31.7</td>
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<td>20.1</td>
<td>9.7</td>
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<td>-6.90</td>
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<td>0.1</td>
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<td>-5.80</td>
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<tr>
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<td>-4.30</td>
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<td>2.34</td>
<td>5.9</td>
<td>8.9</td>
<td>3.0</td>
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</table>

Notes: Statistics are for product 1. $α$ is the price coefficient for free-agent customers in the lock-in model. The price coefficient for locked-in customers is $α + \bar{α}$, where $\bar{α} = 3$. Lock-in Avg. $α$ is the average price coefficient in the market, where the average is weighted by the shares of free-agent and locked-in consumers. Logit $α$ is the calibrated price coefficient for the standard logit model. Bias is post-merger price prediction error measured in percentage points.
Figure 7: Cumulative Pass-through for Common Costs

(a) Unexpected Costs

(b) Expected Costs
Figure 8: Cumulative Pass-through Controlling for Rival Prices

(a) Unexpected Costs

(b) Expected Costs
B.3 Passthrough Heterogeneity: Four-Parameter Specification

Here we report summary statistics for our four-parameter estimates of firm-specific heterogeneity.

Table 18: Pass-through Heterogeneity: Four-Parameter Specification

<table>
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<tr>
<th>Parameter</th>
<th>p10</th>
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<th>p50</th>
<th>p75</th>
<th>p90</th>
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<tbody>
<tr>
<td>Anticipation</td>
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<td>-6.5</td>
<td>-2.2</td>
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<tr>
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<td>0.016</td>
<td>0.025</td>
<td>0.039</td>
<td>0.066</td>
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<tr>
<td>Duration</td>
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<td>16.5</td>
<td>29.0</td>
<td>48.2</td>
<td>60.8</td>
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<tr>
<td>Finish</td>
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<td>12.7</td>
<td>21.9</td>
<td>32.3</td>
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<tr>
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<td>0.532</td>
<td>0.711</td>
<td>0.896</td>
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B.4 Summary Statistics by Brand

Table 19: Summary of Brands

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<th>Brand</th>
<th>Cond. Share</th>
<th>Share</th>
<th>Num. Markets</th>
<th>Num. Stations</th>
<th>Margins</th>
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<td>Speedway</td>
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<td>Loves</td>
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<td>0.01</td>
<td>15</td>
<td>1.00</td>
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<td>Thornton</td>
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<td>35</td>
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</table>

B.5 Elasticities Over Time
Figure 9: Elasticities Over Time

(a) Unaffiliated and Affiliated

(b) Mean Elasticity
Figure 10: Elasticities: Large Brands versus Fringe

(a) Affiliated Elasticity

(b) Unaffiliated Elasticity

(c) Weighted Elasticity